Continuous-time Fourier Transform

- DEVELOPMENTS – SO FAR
  - Complex exponentials are eignfunctions of LTI Systems
  - Therefore, it is desireble to express signal in terms of complex exponentials.
  - We have developed CT FS and DT FS for representation of periodic signals.
  - FS can also be thought as discrete in frequency, that is, it tells us that CT/DT periodic signal is composed of complex exponentials with same period.

  - FS Summary:

	time	frequency
CT	periodic	discrete aperiodic
DT	periodic	discrete periodic

  - As I have mentioned multiple times; (Important Concept)
    - Periodicity in one domain implies discreteness in other domain.
    - OR – Continuous in one domain ____ aperiodicity in ______.

  - Since signals are not periodic in nature in practice, we need to develop representation of aperiodic signals in terms of complex exponentials. This is our topic of interest in next two chapters (Ch4 and Ch5).

  - We begin with CT aperiodic signal.

- CT FOURIER TRANSFORM:
  - We begin with an example before formulation.
  - Let $x(t)$ be aperiodic signal of the form

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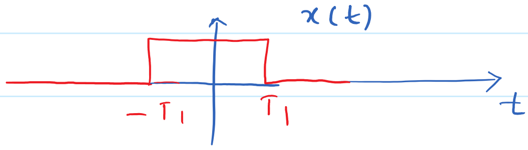
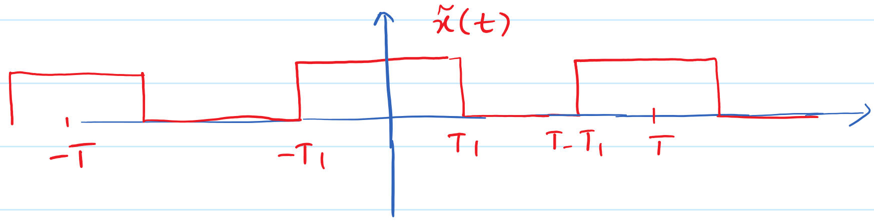


- By taking $T \geqslant 2T_1$, define $\tilde{x}(t)$ as $x(t)$ from $-\frac{T}{2} \le t \le \frac{T}{2}$ and periodic with period T



- $\tilde{x}(t)$ is a periodic signal
- Also, $x(t) = \lim_{T \to \infty} \tilde{x}(t)$

  since $x(t) = \tilde{x}(t)$ for $-\frac{T}{2} \le t \le \frac{T}{2}$

- As $\tilde{x}(t)$ is periodic, we have FS representation for $\tilde{x}(t)$

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

  where

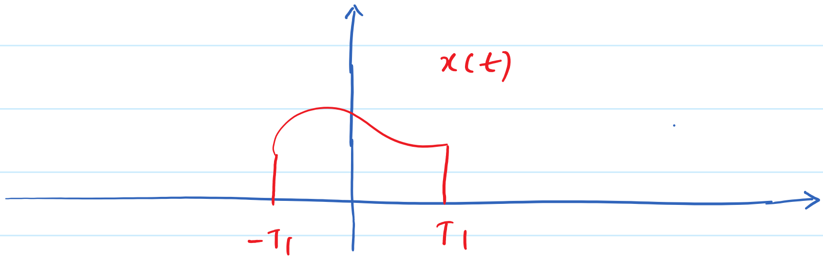
$$a_k = \frac{\sin(k\omega_0 T_1)}{k\pi}$$

- We plot 'a_k' on continuous 'ω' scale for $T_1 = 1$ and different values of T as also shown in Figure (textbook).

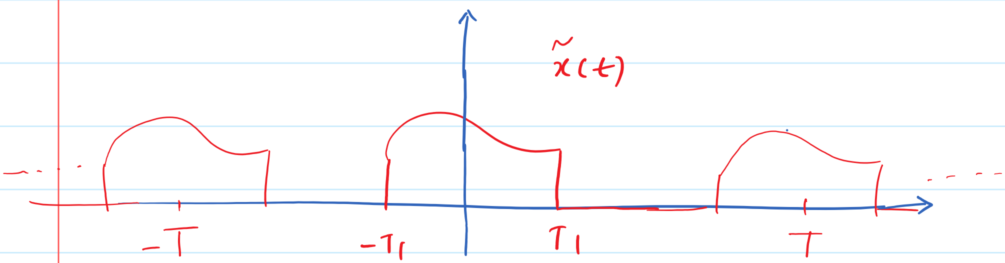
- It can be observed that the frequencies present in the signal $\tilde{x}(t)$ come closer and closer as $T \to \infty$. Consequently, we have continuous range of frequencies for aperiodic signal.

Let's formulate the CT Fourier Transform.

Define $x(t)$ be CT aperiodic signal such that $x(t)=0$ for $|t| > T_1$, that is,



Also define period signal such that $x(t) = \tilde{x}(t)$ for $|t| \le T_1$



Since $\tilde{x}(t)$ is periodic, we have FS representation

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{—} \quad (1)$$

where

$$a_k = \frac{1}{T} \int_{-T/2}^{T/2} \tilde{x}(t) e^{-jk\omega_0 t} dt$$

As $x(t) = \tilde{x}(t)$, $|t| \le T_1 \le \frac{T}{2}$ and $x(t) = 0$, $|t| \ge \frac{T}{2} \ge T_1$

$$\Rightarrow \quad a_k = \frac{1}{T} \int_{-\infty}^{\infty} x(t) e^{-jk\omega_0 t} dt$$

We define, $$X(j\omega) \triangleq \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Clearly, $$a_k = \frac{1}{T} X(jk\omega_0)$$

Substituting this in (1) yields:

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T} X(jk\omega_0)\, e^{jk\omega_0 t}$$

Now; we apply limit as $\lim_{T\to\infty} \tilde{x}(t) = x(t)$

$\tilde{x}(t)$ → Periodic, $x(t)$ → Aperiodic

Also, substitute $T = \frac{2\pi}{\omega_0}$

$$\lim_{T\to\infty} \equiv \lim_{\omega_0 \to 0}$$

$$\Rightarrow x(t) = \lim_{T\to\infty} \tilde{x}(t) = \frac{1}{2\pi} \lim_{\omega_0\to 0} \sum X(jk\omega_0)\, e^{jk\omega_0 t}\, \omega_0$$

$$\Rightarrow x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

This is the synthesis equation for aperiodic signal. This informs us that all (continuous range) of frequencies contribute to the aperiodic signal.

Analysis Equation: $X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

- $X(j\omega) \rightarrow$ Fourier transform (FT) of the signal $x(t)$.
- $X(j\omega) \rightarrow$ signal representation in ω (frequency) domain.
- $X(j\omega) \rightarrow$ Quantifies the contribution of $e^{j\omega t}$ in the signal.

Examples :-

Example 1 : (4.1)

Find FT of $x(t) = e^{-\alpha t} u(t)$, $Re\{\alpha\} > 0$

Use analysis Equation:

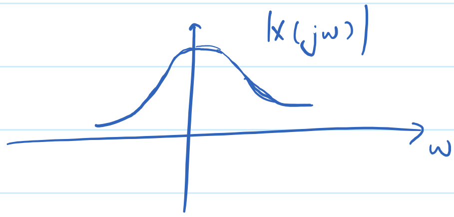
$$X(jw) = \int_{0}^{\infty} e^{-\alpha t} e^{-jwt} \, dt$$

because of $u(t)$.

$$X(jw) = - \frac{e^{-(\alpha + jw)t}}{\alpha + jw} \Bigg|_{0}^{\infty} = \frac{1}{\alpha + jw}$$

Sine $X(jw)$ is complex, we plot magnitude and phase of $X(jw)$ separately.

$$|X(jw)| = \frac{1}{\sqrt{\alpha^2 + w^2}}$$



w	$\lvert X(jw) \rvert$
0	$1/a$
a	$\frac{1}{\sqrt{2}\, a}$
⋮	

Interpretation: Signal $x(t)$ has more low-frequency content.

$$\measuredangle X(jw) = - \tan^{-1}(w/a)$$

Example 2 : (4.2)

Find FT $Y(jw)$ of the signal $y(t) = e^{-\alpha |t|}$, $Re\{\alpha\} > 0$

→ Obviously, we can use analysis equation (textbook)

→ But we can reuse the result of Example 1.

→ We have time-reversal property:

$$FT \text{ of } x(t) \longrightarrow X(j\omega)$$
$$FT \text{ of } x(-t) \longrightarrow X(-j\omega)$$

– We note first;

$$y(t) = x(t) + x(-t)$$

for $x(t)$ in example 4.1

$$\Rightarrow \quad Y(j\omega) = \frac{1}{\alpha + j\omega} + \frac{1}{\alpha - j\omega} = \frac{2\alpha}{\alpha^2 + \omega^2}$$

Example 3 (4.3)

For $x(t) = \delta(t)$, find FT.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt$$

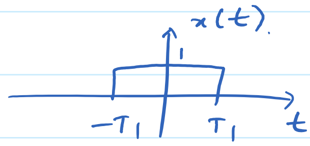
$$= e^{-j\omega(0)} = \boxed{1}$$

Interpretation:

Complex exponentials of all frequencies contribute equally to form an impulse function.

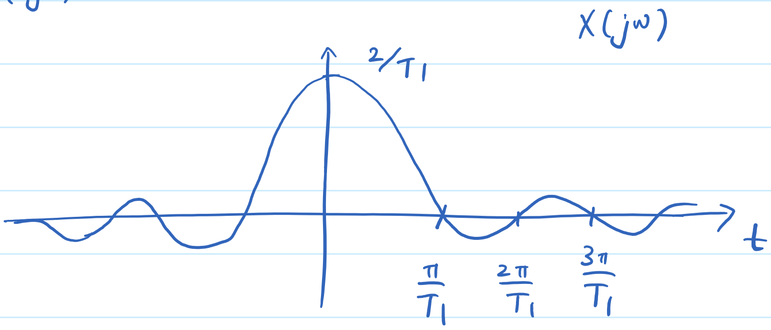
Example 4 : (4.4)

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| > T_1 \end{cases}$$



$$X(j\omega) = \int_{-T_1}^{T_1} e^{-j\omega t}\, dt = \frac{2\sin\omega T_1}{\omega}$$

$X(j\omega) \longrightarrow$ Sinc.



We also covered

- the inverse FT of impulse signal in frequency domain and shifted impulse signal in frequency domain.
- Fourier transform of CT periodic signals using the result of inverse Fourier transform of Impulse function in frequency domain.

Continuous-time Fourier Transform - Examples (contd.)

- Recap : Fourier Transform

  - Fourier Transform (FT) of a signal $x(t)$ is defined as

Analysis Equation: $$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

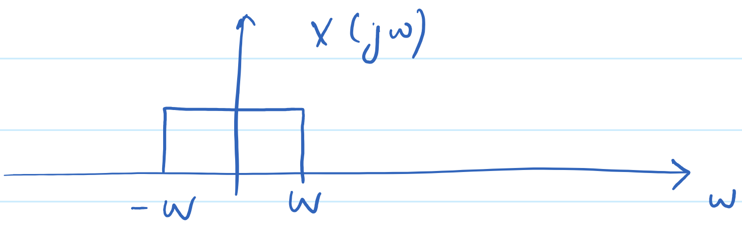
  - Using FT, signal can be synthesized as

Synthesis Equation: $$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

- Examples
  - Covered in last lecture: Given $x(t)$. Find $X(j\omega)$.
  - What about if we have $X(j\omega)$ and we want to find time domain signal.

Example

Given $$X(j\omega) = \begin{cases} 1 & |\omega| < W \\ 0 & |\omega| > W \end{cases}$$



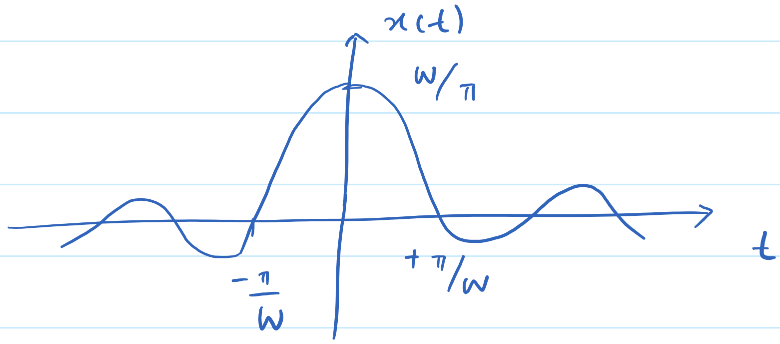
- First interpret $X(j\omega)$; It look like the FT of ideal Low-Pass Filter.

- What is a Low-Pass Filter? A system which allows low frequency signals to pass through and stops high frequency signals. Here 'W' is known as cut-off frequency of the filter.

- Comeback to problem: find $x(t)$.
- Use synthesis equation.

$$x(t) = \frac{1}{2\pi}\int_{-W}^{W} e^{j\omega t}\,d\omega$$

$$= \frac{1}{2\pi}\,\frac{e^{jWt} - e^{-jWt}}{jt} = \frac{\sin Wt}{\pi t}$$

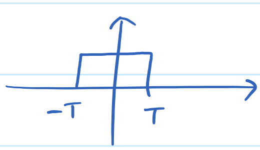


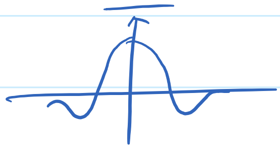
Time to reflect:

t

w

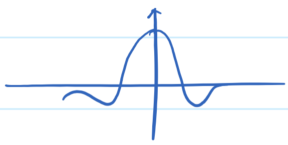
$x(t)$:

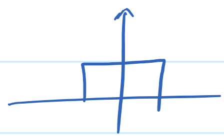




$X(j\omega) = \frac{2\sin\omega T_1}{\omega}$

$x(t)$





$X(j\omega) = \frac{\sin Wt}{\pi t}$

- Rectangular Pulse in one domain $\longleftrightarrow$ sinc in other
- We are moving towards duality; interchange 't' and 'w'; shape/behaviour remains same

- We will formally define duality later.

- In text book(s), sinc is defined as

$$\text{sinc}(\theta) = \frac{\sin(\pi\theta)}{\pi\theta}.$$

Using this definition; we can express

$$\frac{2\sin\omega T_1}{\omega} = 2T_1 \,\text{sinc}\left(\frac{\omega T_1}{\pi}\right)$$

$$\frac{\sin(Wt)}{\pi t} = \frac{W}{\pi}\,\text{sinc}\left(\frac{Wt}{\pi}\right)$$

Example: $X(j\omega) = 2\pi\,\delta(\omega-\omega_0)$

$$x(t) = \frac{1}{2\pi}\int 2\pi\,\delta(\omega-\omega_0)\,e^{j\omega t}\,d\omega$$

$$\boxed{x(t) = e^{j\omega_0 t}}$$

Remarkable
Find its dual?

- Convergence of FT:

  - We have similar two set of conditions

1st:

$$\int_{-\infty}^{\infty} |x(t)|^2 < \infty$$

($x(t)$ Absolute Square Integrable)

2nd:

1- $$\int_{-\infty}^{\infty} |x(t)|\,dt < \infty$$

($x(t)$ Absolute Integrable)

1 - $\int |x(t)| dt < \infty$ (Absolute Integrable)

2 - Finite no. of minima and maxima

3 - Finite no. of discontinuities.

Properties of CT Fourier Transform

Shorthand; $x(t) \xleftrightarrow{FT} X(jw)$

$y(t) \xleftrightarrow{FT} Y(jw)$

- Linearity :

$$\alpha x(t) + \beta y(t) \longleftrightarrow \alpha X(jw) + \beta Y(jw)$$

- Obvious; by definition.

- Time shift

$$y(t) = x(t-t_0)$$

$$Y(jw) = e^{-jwt_0} X(jw)$$

Proof; (very straighforward)

$$Y(jw) = \int y(t) e^{-jwt} dt$$

$$= \int x(t-t_0) e^{-jwt} dt$$

Let $t - t_0 = \tau$

$$= e^{-jwt_0} \int x(\tau) e^{-jw\tau} d\tau$$

$$Y(jw) = e^{-jwt_0} X(jw) \qquad \text{QED.}$$

$$Y(j\omega) = e^{-j\omega t_0} X(j\omega) \qquad \text{QED.}$$

- Conjugation Symmetry:

$$y(t) = x^*(t)$$

$$Y(j\omega) = X^*(-j\omega)$$

(Do you remember $a_k = a^*_{-k}$ Something Similar)

- Time / Frequency scaling

$$y(t) = x(\alpha t)$$

$$Y(j\omega) = \frac{1}{|\alpha|} X\left(\frac{j\omega}{\alpha}\right)$$

- Special case; $\alpha = -1$ (Reversal)

$$y(t) = x(-t)$$

$$\Rightarrow \quad Y(j\omega) = X(-j\omega)$$

- Reflect on scaling property; Develop link with rectangular pulse example.

Properties of CT Fourier Transform (Contd.)

- DIFFERENTIATION IN TIME/FREQUENCY PROPERTY

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$\frac{dx(t)}{dt} \quad x'(t) \xleftrightarrow{FT} j\omega X(j\omega) \quad \text{(Time-derivative)}$$

$$-jt\,x(t) \xleftrightarrow{FT} X'(j\omega) = \frac{d}{d\omega}\left(X(j\omega)\right) \quad \text{(Freq. derivative)}$$

Proof: very obvious!

Before we present integration property, we review one example and derive convolution property.

<u>Example :-</u>

- Consider a signal $g(t) = u(t)$. We want to find FT using inverse of differentiation.

We know,

$$g'(t) = \delta(t)$$

$$\Rightarrow \quad g'(t) \longleftrightarrow 1 = j\omega\, G(j\omega).$$

$$\Rightarrow \boxed{G(j\omega) = \frac{1}{j\omega}}$$

- Now consider a signal $z(t) = u(t) - \frac{1}{2}$

$$z'(t) = \delta(t)$$

$$\Rightarrow \quad z'(t) \xleftrightarrow{FT} 1 = j\omega\, Z(j\omega)$$

$$\Rightarrow \boxed{Z(j\omega) = \frac{1}{j\omega}}$$

This is bizarre! We have two different signals and we are getting same FT using inverse of differentiation property, that is,

$$g(t) = u(t) \qquad\qquad Z(t) = u(t) - \frac{1}{2}$$

$$\updownarrow \qquad\qquad\qquad \updownarrow$$

$$G(j\omega) = \frac{1}{j\omega} \qquad\qquad Z(j\omega) = \frac{1}{j\omega}$$

Q; Which one is correct OR which time domain signal correspond to FT $\frac{1}{j\omega}$?

A; $z(t)$

Q; Why ?

A; Because it has zero average (DC) value. $g(t)$ does not have DC value, which gets lost when we take derivative.

So; FT of $z(t)$, $Z(j\omega) = \frac{1}{j\omega}$

and FT of $g(t)$, $G(j\omega) \neq \frac{1}{j\omega}$

Q; So, what is FT of $g(t)$?

we relate $g(t)$ and $z(t)$ as

$$g(t) = z(t) + \frac{1}{2}$$

$$\Rightarrow \quad G(j\omega) = Z(j\omega) + \frac{1}{2} 2\pi \delta(\omega)$$

$$\Rightarrow \quad G(j\omega) = Z(j\omega) + \frac{1}{2} 2\pi \delta(\omega)$$

$$\Rightarrow \quad \boxed{G(j\omega) = \frac{1}{j\omega} + \underline{\pi \delta(\omega)}}$$

we have this term because of non-zero DC (average) value of $g(t)$.

- CONVOLUTION PROPERTY:
  - The most remarkable property; Tons of Implications
  - One of the major reasons, we move from time to frequency domain.

PROPERTY.

$$x(t) \xleftrightarrow{FT} X(j\omega) \qquad y(t) \xleftrightarrow{FT} Y(j\omega)$$

$$x(t) * y(t) \xleftrightarrow{FT} X(j\omega) Y(j\omega)$$

<u>PROOF</u>; Let, $z(t) = x(t) * y(t)$

By definition; $Z(j\omega) = \int z(t) e^{-j\omega t} dt$

$$Z(j\omega) = \int \int x(\tau) y(t-\tau) d\tau \, e^{-j\omega t} dt$$

Using $e^{-j\omega t} = e^{-j\omega t + j\omega\tau - j\omega\tau}$, and rearranging terms/integrals.

$$Z(j\omega) = \int_{-\infty}^{\infty} x(\tau) e^{-j\omega\tau} \left(\int_{-\infty}^{\infty} y(t-\tau) e^{-j\omega(t-\tau)} dt \right) d\tau$$

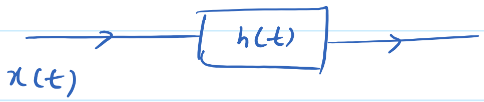
$$= \int_{-\infty}^{\infty} x(\tau) e^{-j\omega\tau} d\tau \quad Y(j\omega)$$

$$Z(j\omega) = X(j\omega) Y(j\omega)$$, QED (Probably)

- Using convolution property, we can find impulse responses of different systems.

Examples

Ex 1: Impulse response of time-delay system



$y(t) = x(t-t_0)$

We know $h(t) = \delta(t-t_0)$; which we determine here using FT properties.

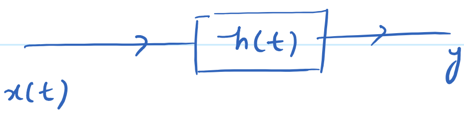
Time shift property; $\quad Y(j\omega) = e^{-j\omega t_0} X(j\omega)$

Convolution ——— ; $\quad Y(j\omega) = H(j\omega)\, X(j\omega)$

$$\Rightarrow \quad H(j\omega) = e^{-j\omega t_0}$$

$$\Rightarrow \quad h(t) = \delta(t-t_0)$$

Example 2; (Differentiator System)



$y(t) = \frac{d}{dt} x(t)$

$H(j\omega) = ?$

$$Y(j\omega) = X(j\omega)\; j\omega \quad \Rightarrow \quad H(j\omega) = j\omega.$$

$\boxed{H(j\omega) = j\omega}$ has very cool interpretation!

. Differentiator boosts / amplifies high frequency components.

Example 3 (Application of Convolution Property)

Given $x(t) = e^{-\alpha t} u(t)$ input to LTI system with impulse respone $h(t) = e^{-\beta t} u(t)$, determine output $y(t)$ a system

We know, $y(t) = x(t) * h(t)$ (Tedious job!)

Alternatively $Y(j\omega) = X(j\omega) \; H(j\omega)$

$$\Rightarrow \quad Y(j\omega) = \frac{1}{\alpha + j\omega} \; \frac{1}{\beta + j\omega}$$

• To find time-domain signal $y(t)$; we use partial fraction approach.

Case 1; $\alpha \neq \beta$

$$\Rightarrow \quad Y(j\omega) = \frac{A}{\alpha + j\omega} + \frac{B}{\beta + j\omega}$$

$$\Rightarrow \quad A = \frac{1}{\beta - \alpha}, \qquad B = -\frac{1}{\beta - \alpha}$$

Take inverse FT of $Y(j\omega)$:

$$\Rightarrow \quad y(t) = A\, e^{-\alpha t} u(t) - B e^{-\beta t} u(t)$$

$\rightarrow$ Easier than convolution.

Case 2: $\alpha = \beta$

$$\Rightarrow \quad Y(j\omega) = \frac{1}{(\alpha + j\omega)^2}$$

How to find $y(t)$?

we know

$$e^{-\alpha t} u(t) \xleftrightarrow{FT} \frac{1}{\alpha + j\omega}$$

$$-jt\, e^{-\alpha t} u(t) \xleftrightarrow{FT} \frac{d}{d\omega}\left(\frac{1}{\alpha + j\omega}\right)$$

$$= \frac{-j}{(\alpha + j\omega)^2}$$

$$\Rightarrow \quad t\, e^{-\alpha t} u(t) \xleftrightarrow{FT} 1$$

$$\overline{(\alpha + j\omega)^2}$$

$$\Rightarrow \quad t e^{-\alpha t} u(t) \xleftrightarrow{FT} \frac{1}{(\alpha + j\omega)^2}$$

$$\Rightarrow \quad y(t) = \begin{cases} t e^{-\alpha t} u(t) & \alpha = \beta \\ \frac{1}{\beta - \alpha}\left[e^{-\alpha t} u(t) - e^{-\beta t} u(t)\right] & \alpha \neq \beta \end{cases}$$

- INTEGRATION PROPERTY

- We derive it using convolution property and result of FT of $u(t)$.
- We want to find FT of $\int_{-\infty}^{t} x(\tau)\, d\tau$

- Short-hand notation $F\{x(t)\} = X(j\omega)$.
- We know; $\int_{-\infty}^{t} x(\tau)\, d\tau = x(t) * u(t)$

$$\Rightarrow F\left\{\int_{-\infty}^{t} x(\tau)\, d\tau\right\} = F\{x(t)\}\, F\{u(t)\}$$

$$= X(j\omega)\left(\frac{1}{j\omega} + \pi\, \delta(\omega)\right)$$

$$\Rightarrow F\left\{\int_{-\infty}^{t} x(\tau)\, d\tau\right\} = \frac{X(j\omega)}{j\omega} + \pi\, X(0)\, \delta(\omega).$$

Integration Property.

- DUALITY:

- We have been using this term for a while.
- CT signal and its FT are dual of each other.
- Let's formally define this;

If $\quad x(t) \xleftrightarrow{FT} X(j\omega)$

$y(t) = X(jt) \xleftrightarrow{FT} ?\ Y(j\omega)$

If $\quad x(t) \longleftrightarrow X(j\omega)$

$$y(t) = X(jt) \xleftrightarrow{FT} \; ? \; Y(j\omega)$$

In words, $y(t) = X(jt)$, Replace ω with t in $X(j\omega)$

Now;

$$Y(j\omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} X(jt)\, e^{-j\omega t}\, dt \quad \text{— } 1$$

Now, write Synthesis equation:

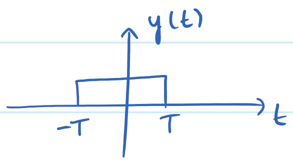
$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$\Rightarrow \quad 2\pi\, x(-t) = \int_{-\infty}^{\infty} X(j\omega)\, e^{-j\omega t}\, d\omega \quad \text{— } 2$$

Comparing right hand-sides of 1 and 2

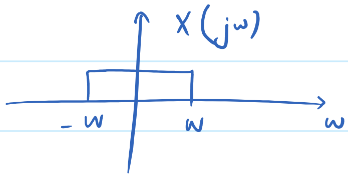
$$\Rightarrow \quad \boxed{Y(j\omega) = 2\pi\, x(-t)}$$

Evidence;



$$\frac{2\sin T\omega}{\omega} = Y(j\omega)$$

$$x(t) = \frac{\sin Wt}{\pi t}$$



Since $\quad y(t) = X(jt)$ with $T = W$

$$\Rightarrow \quad Y(j\omega) = 2\pi\, x(-t) \quad \text{with } T = W$$

MULTIPLICATION PROPERTY

$$F\{x(t)\} = X(j\omega)$$

$$F\{y(t)\} = Y(j\omega)$$

Property: $$F\{x(t)\,y(t)\} = \frac{1}{2\pi}\left(X(j\omega) * Y(j\omega)\right)$$

Proof:

Let $$Z(j\omega) = \frac{1}{2\pi}\left(X(j\omega) * Y(j\omega)\right)$$

By synthesis equation,

$$z(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Z(j\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\,\frac{1}{2\pi}\int_{\omega=-\infty}^{\infty}\int_{\eta=-\infty}^{\infty} X(j\eta)\, Y(j(\omega-\eta))\, d\eta\; e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} X(j\eta)\, e^{+j\eta t}\, d\eta\;\; \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j(\omega-\eta))\, e^{j(\omega-\eta)t}\, d\omega$$

$$z(t) = x(t)\; y(t)$$ QED

Quite easily done!

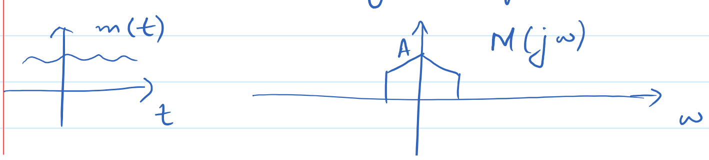
- Applications of Multiplication Property.
  - Amplitude Modulation (AM)

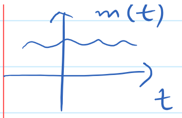
System Block Diagram:

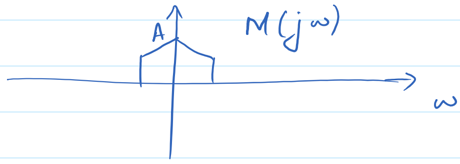
Transmitter

Receiver

- $m(t)$ = Message signal (Low frequency)



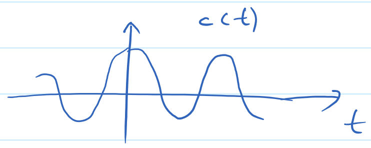


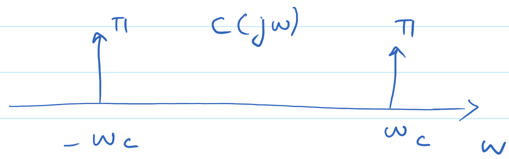


- Since $m(t)$ is low frequency, we cannot transmit it

- We modulate $m(t)$ with carrier signal $c(t)$ to make it high frequency.

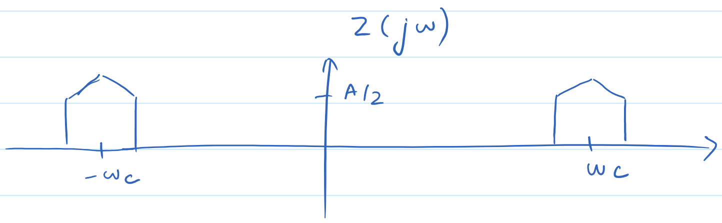
- $c(t) = \cos(\omega_c t)$ $\omega_c \rightarrow$ carrier frequency





- Transmitted Signal $z(t) = m(t)\, c(t)$

By multiplication property:



Mathematically;

$$C(j\omega) = \pi\, \delta(\omega - \omega_c) + \pi\, \delta(\omega + \omega_c)$$

$$Z(j\omega) = \frac{1}{2\pi}\, M(j\omega) * C(j\omega)$$

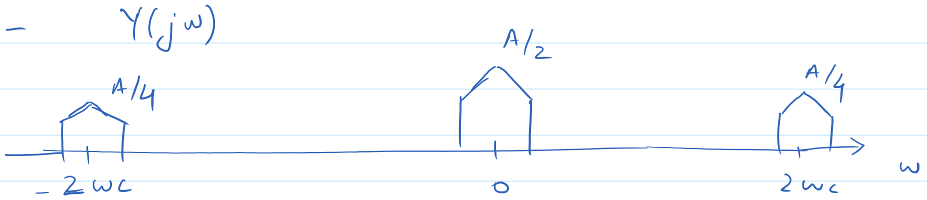
$$= \frac{1}{2\pi}\, M\big(j(\omega - \omega_c)\big) + M\big(j(\omega + \omega_c)\big)$$

<u>Receiver Side</u> :-

- Received signal $z(t)$

- Let $y(t) = z(t)\, c(t)$

- $Y(j\omega)$



- LPF with magnitude 2 to obtain recovered signal $r(t) = m(t)$.

- Another Application: Frequency Selective Filtering
  - How to use low-pass filter as band-pass filter
  - Answer; pre-modulate and post-modulate
  - See 4.5.1 (To be covered in the lecture briefly)

We also covered use of Fourier Transform for analysis of LTI systems described by Linear Constant Coefficient (LCC) differential equations. See relevant section in the book and examples.