Lahore University of Management Sciences
Department of Electrical Engineering, Syed Babar Ali School of Science and Engineering

EE310 - Signals and Systems

Spring 2019 – March 29, 2019

Mid-term Exam

Time Allowed: 150 minutes | **Total Points**: 100

Student ID ____________________________ Signature ________________________________

Instructions

1. Do not flip this page over until told to do so.
2. The exam is closed-book, closed-notes. A two-sided hand-written A4-sized formula sheet is allowed. Calculators are also allowed.
3. Try to identify the easiest way to solve a problem.
4. Clearly outline all your steps. Solutions with inadequate justifications and/or steps may not receive full credit.
5. **The exam consists of TWO Parts.**
   (a) The first part is worth 40 pts, and is to be solved on the exam sheet. It is to be returned to the exam staff in the first 60 minutes.
   (b) The second part is worth 60 pts and is also to be solved on the exam sheet. Blue answer books are only provided for rough work. Do not return them after the exam.
6. **Note**: CT and DT refers to continuous-time and discrete-time, respectively.

Problem 1	[40]	
Problem 2	[15]	
Problem 3	[10]	
Problem 4	[15]	
Problem 5	[10]	
Problem 6	[10]	
Total	[100]	

EE310 – Signals and Systems
Midterm Examination Spring 2019

Part - 1 - [Solutions] (40 pts)

Student ID ____________________________ Name ________________________________

Section Instructor ____________________ Signature ________________________________

Instructions: This part needs to be solved on this sheet, and is to be returned to the exam staff in the first 60 minutes.

Problem 1. Select ALL (upto three) correct choices. Every incorrect answer would earn a penalty of 2/3 points but the total marks of any multiple choice question will not be less than zero.

(1) For continuous-time (CT) signals, choose the false statement(s):

(a) A CT signal which is periodic with period 2π is also periodic with period 4π .
(b) [The sum of two periodic CT signals of different periods is always periodic.]
(c) All periodic CT signals have a Fourier Series representation.
(d) [The sum of two non-periodic CT signals is never periodic.]

(2) Consider the series/cascade connection of the two CT LTI systems with impulse responses denoted by $h_1(t)$ and $h_2(t)$, Which of the following statements are true?

(a) $[y(t) = x(t) * h_1(t) * h_2(t)]$
(b) $[y(t) = x(t) * h_2(t) * \delta(t) * h_1(t)]$
(c) $y(t) = x(t) * \big(h_1(t) \times h_2(t)\big)$
(d) $[y(t) = h_1(t) * \delta(t-1) * x(t) * \delta(t+1) * h_2(t)]$

(3) For the discrete-time signal $x[n] = \sin(n/16)$, which of the following is/are true.

(a) The Fourier series of the signal has only one non-zero term.
(b) The Fourier series of the signal has two non-zero terms.
(c) [The signal does not have Fourier series.]
(d) The Fourier series of the signal has 16 non-zero terms.

(4) Which of the following represent the impulse response of a *stable* LTI system?

(a) $[h[n] = u[n] - u[n-10]]$
(b) $h[n] = u[-n-10] - u[n-10]$
(c) $[h[n] = \delta[n]]$
(d) $h[n] = 1.01^n u[n]$

(5) What is the even part of $\delta(t)$?
$\Big($*Recall*: Even part of a signal $x(t)$ is equal to $\frac{1}{2}x(t) + \frac{1}{2}x(-t)$.$\Big)$

(a) $[\frac{d}{dt}\big(u(t)\big)]$
(b) $0.5\delta(t)$
(c) $2\delta(t)$
(d) $[\delta(t)]$

(6) Choose the correct statement(s) for the continuous-time system described by the following input-output relationship $y(t) = x(t)u(-t)$.

(a) The system is non-linear
(b) [The system is stable]
(c) The system is non-causal
(d) [The system is memoryless]

(7) For the continuous-time periodic signal shown below:

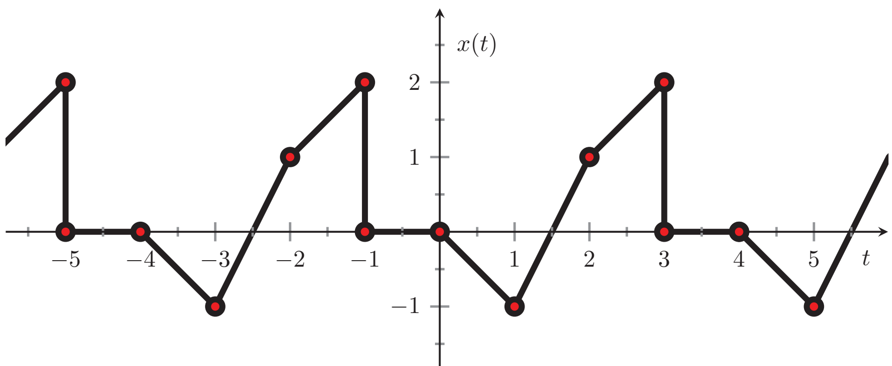


Figure 1: Real CT Periodic Signal $x(t)$ with Fundamental Period $T = 4$

What is the DC value of the signal or the Fourier series coefficient a_0?

(a) -1/2
(b) 0
(c) 2
(d) [1/4]

(8) Consider the periodic continuous-time signal $x(t)$, which is the absolute value of the real-valued signal in previous problem. (For example it goes through the point $(1, 1)$ instead of $(1, -1)$ and still passes through $(1.5, 0)$). Which of the following is/are true?

(a) $|x(t)|$ has same Fourier series coefficients as of $x(t)$.
(b) [$|x(t)|$ is a power signal]
(c) $|x(t)|$ has the same DC value as $x(t)$
(d) [$|x(t)|$ has the same power as $x(t)$]

(9) What is the fundamental period of continuous-time signal $2\cos(t + \pi/4) + 4\sin(\pi t/3)$?

(a) 3
(b) 2π
(c) 6π
(d) [Undefined, the signal is not periodic and has no fundamental period.]

(10) What is the *fundamental* period of the DT periodic signal

$$x[n] = (-1)^{3n}\cos(\pi n/3)?$$

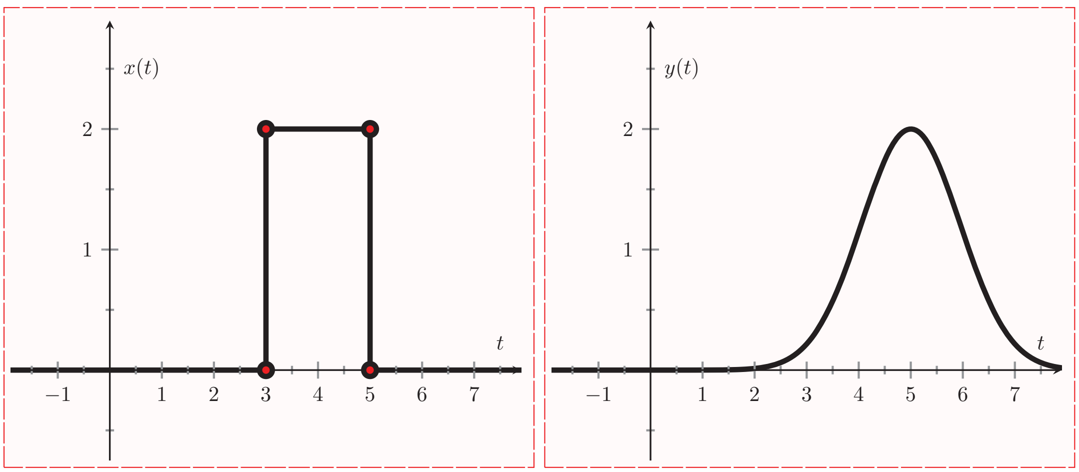
(a) 3
(b) [6]
(c) 2
(d) 12

(11) Choose the correct statement(s) for the continuous-time signal

$$x(t) = \sin(\pi t/3) - 2\cos(\pi t/4 - \pi/5).$$

(a) [$x(t)$ has fundamental frequency given by $\omega_o = \pi/12$.]
(b) [$x(t)$ is periodic.]
(c) The only non-zero Fourier coefficients of $x(t)$ are a_1 and a_{-1}.
(d) [$x(t)$ has zero DC ($k = 0$ Fourier series) component.]

(12) Suppose $x(t)$ given below (left) is the input signal to a continuous-time system and $y(t)$ given below (right) is the output. Which of the following is/are correct about the causality of the system?

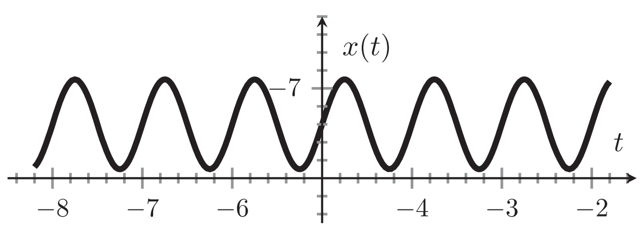


(a) $x(t)$ (b) $y(t)$

Figure 1: Input $x(t)$ and output $y(t)$ for some continuous-time system.

(a) The system is causal.
(b) The system is non-causal
(c) The system is causal only if it is LTI.
(d) [The system could be causal or non-causal because there is not enough information.]

(13) Choose the correct statement(s) for the continuous-time periodic signal (offset sinusoidal) drawn below



(a) The DC component value is zero.
(b) The DC component value is strictly greater than zero.
(c) [The number of non-zero spectral coefficients of the signal are 3.]
(d) The number of non-zero spectral coefficients of the signal are 2.

(14) Consider the system described by $y(t) = \big(x(t+1)\big)^*$, where $x(t)$ is the complex input, $y(t)$ is the complex output and $(\cdot)^*$ denotes complex conjugation. The system is

(a) [Non-linear.]
(b) Causal.
(c) [Non-causal.]
(d) Time-varying.

(15) Let $x(t)$ be a continuous-time periodic signal with a fundamental period T. Let $y(t) = x(t-2)$. Then $y(t) = y(t+P)$ for

(a) [$P = T$.]
(b) $P = T + 2$.
(c) [$P = 2T$.]
(d) [$P = -T$.]

(16) Let $\delta(t)$ be the CT unit impulse function. Which of the following is/are true?

(a) [$\displaystyle\int_{-\infty}^{-1} \delta(t)\, dt = 0$]
(b) $\displaystyle\int_{-\infty}^{4} \delta(t-1)\, dt = 0$
(c) $x(t)\,\delta(t-t_0) = x(t-t_0)\,\delta(t-t_0)$
(d) [$\displaystyle\int_{-\infty}^{\infty} \delta(t)\, dt = 1$]

(17) The response of an LTI system to the input $2\delta[n-1]$ is an output $g[n]$. What is the impulse response $h[n]$ of the system?

(a) $h[n] = 2g[n-1]$
(b) $h[n] = 0.5\delta[n+1]$
(c) [$h[n] = 0.5g[n+1]$]
(d) $h[n] = 2\delta[n+1]$

(18) If a signal $x[n]$ is non zero only in the range of $-5 \le n \le 5$ and another signal $h[n]$ is non zero in the range of $-7 \le n \le -2$, then the signal $y[n] = x[n] * h[n]$ would

(a) be contained in the region $-7 \le n \le 5$
(b) [be contained in the region $-12 \le n \le 3$]
(c) have non zero values when $n+5 \ge -7$ and $n+5 \le -2$
(d) [have non zero values when $n+5 \ge -7$ and $n-5 \le -2$]

(19) Consider a discrete time periodic signal $x[n] = \cos(\frac{2\pi}{3}n)$. The Fourier series expansion for a period $N = 9$ and $w_0 = \frac{2\pi}{N}$ would have

(a) $a_1 = a_{-1} = \frac{1}{2}$
(b) [$a_3 = a_{-3} = \frac{1}{2}$]
(c) [$a_{-6} = \frac{1}{2}$]
(d) [$a_6 = a_{12} = \frac{1}{2}$]

(20) Suppose the continuous-time signal $x(t) = e^{j5t}$ is input to a continuous-time LTI system with impulse response $h(t)$ and frequency response at $\omega_o = 5$ given by $H(j5) = \sqrt{3} - j$. The convolution of $x(t)$ and $h(t)$ is given by

(a) [$2e^{j(5t-\pi/6)}$]
(b) $5(\sqrt{3}-1)$
(c) $(\sqrt{3}+1)e^{j5t}$
(d) $(\sqrt{3})e^{j(5t-1)}$

Part - 2 (60 pts)

Problem 2. (**15 pts**)

(a) ($3 \times 2 = 6$ **pts**) For each of the following systems described by the input-output relationships with $x[n]$ (or $x(t)$) and $y[n]$ (or $y(t)$) denoting the input signal and output signal respectively, determine whether the system is linear and causal. *Briefly* justify your answer. No credit will be given without correct justification.

System 1: $y(t) = \int_{t-10}^{t} x(\tau - 2) d\tau$

Property	Yes/No	Justification
Linear	Yes	Integral
Causal	Yes	$y(t) = \int_{t-12}^{t-2} x(p) dp$

System 2: $y[n] = \cos\Big(\pi(n+1)\Big) x[n]$

Property	Yes/No	Justification
Linear	Yes	
Causal	Yes	

(b) (5 **pts**) Consider a power signal

$$x(t) = 1 + \frac{1}{2}e^{j\frac{2\pi}{3}t}$$

Compute the signal's power P_∞. Recall that

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} |x(t)|^2 \, dt$$

Solution: You can obviously compute the power in time-domain by first computing $|x(t)|^2$ as follows

$$|x(t)|^2 = (1 + \frac{1}{2}e^{j\frac{2\pi}{3}t})(1 + \frac{1}{2}e^{j\frac{-2\pi}{3}t}).$$

Alternatively, the power can be computed using Parseval's relation. Clearly, $x(t)$ is periodic with fundamental frequency $2\pi/3$. It has two non-zero Fourier series components: $a_0 = 1$ and $a_1 = \frac{1}{2}$. Using Parsevals' relation, P_∞ (in Watts) can be computed as

$$P_\infty = |a_0|^2 + |a_1|^2 = 1 + \frac{1}{4} = 1.25.$$

(c) (4 **pts**) Let $x(t) = e^t \Big[u(t+4) - u(t-2)\Big]$. Sketch $y(t) = x(-2t+1)$.

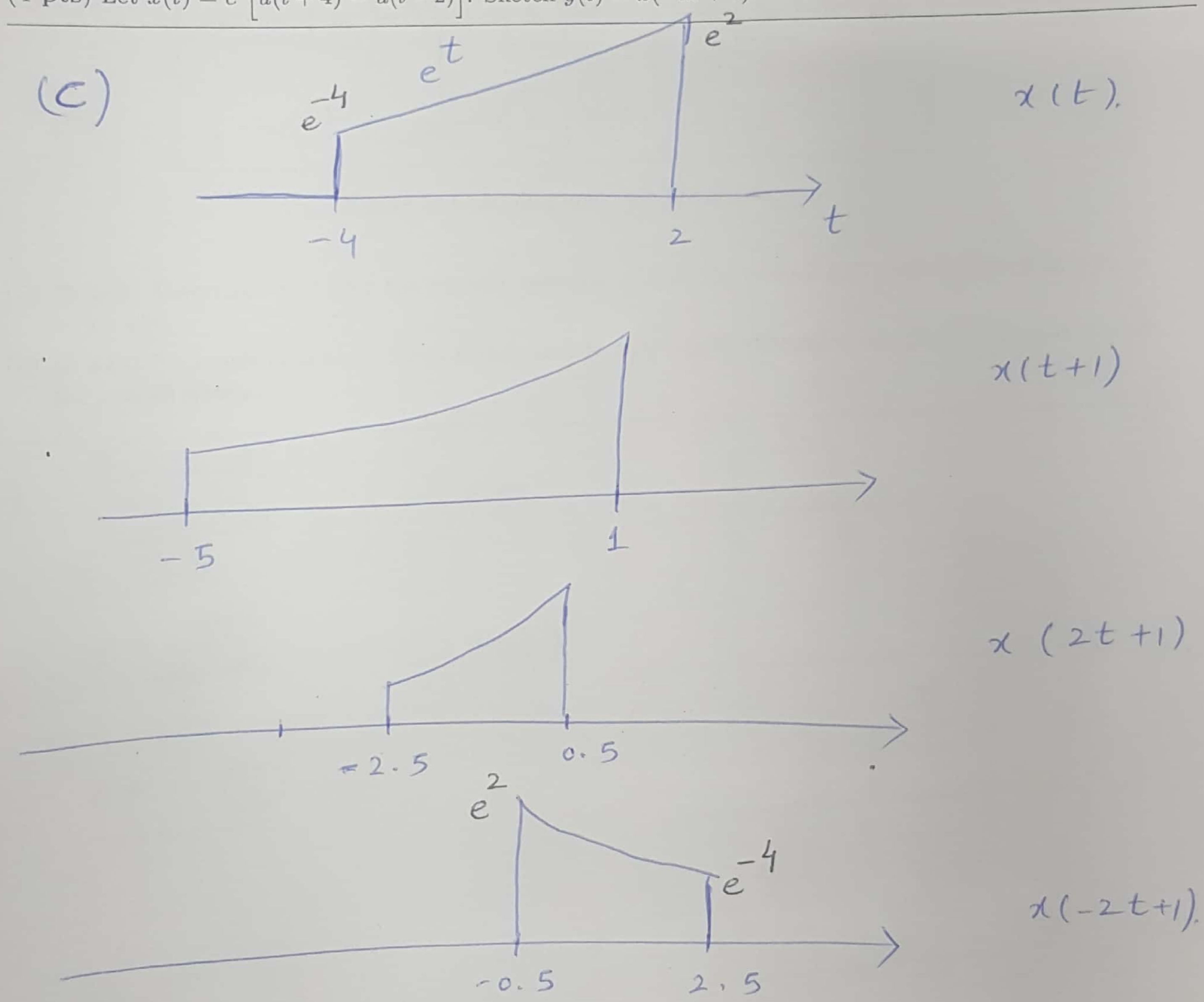
(b) **(5 pts)** Consider a power signal

$$x(t) = 1 + \frac{1}{2} e^{j\frac{2\pi}{3}t}$$

Compute the signal's power P_∞. Recall that

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} |x(t)|^2 \, dt$$

(c) **(4 pts)** Let $x(t) = e^t \Big[u(t+4) - u(t-2) \Big]$. Sketch $y(t) = x(-2t+1)$.



Problem 3. **(10 pts)** Let four continuous time **LTI** systems be connected in a configuration shown in Fig. 2. The impulse response of each system is given by:

$$h_1(t) = u(t-3) - u(t-5)$$
$$h_2(t) = \delta(t+2), \quad h_3(t) = \delta(t-2), \quad h_4(t) = \delta(t+5)$$

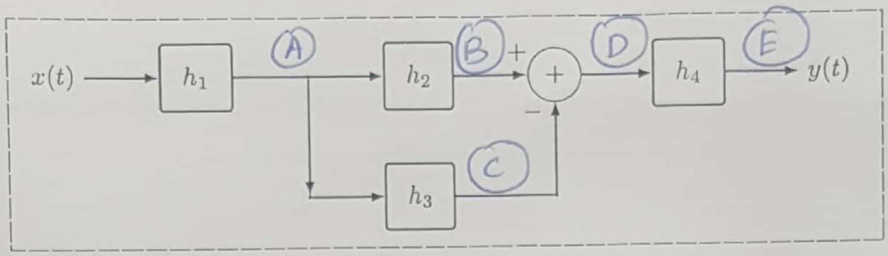


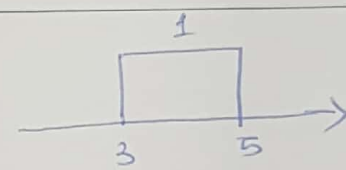
Figure 2: Cascade connection of systems.

(a) **(5 pts)** Determine and plot the impulse response $h(t)$ of the overall **LTI** system such that $y(t) = x(t) * h(t)$.

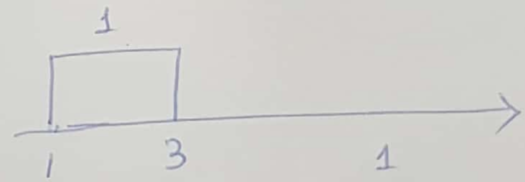
(b) **(5 pts)** For a unit step as an input signal, that is, $x(t) = u(t)$, determine and plot the output $y(t)$ of the overall system.

$x(t) = \delta(t)$

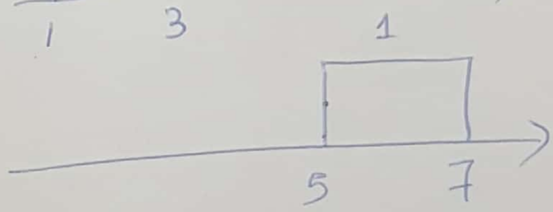
(a) At A : $h_1(t)$



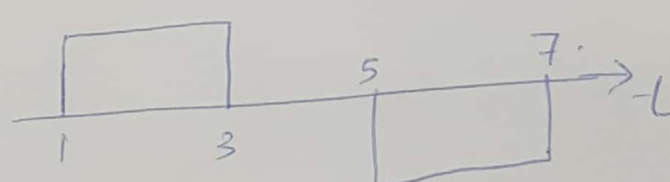
At B $h_1(t+2)$



At C $h_1(t-2)$

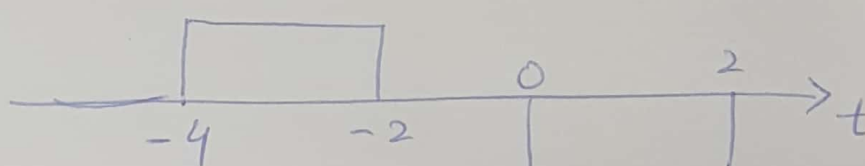


At D

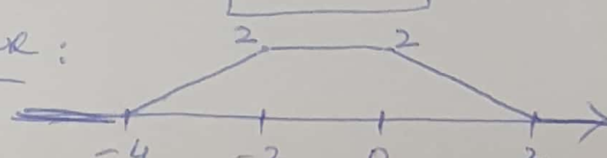


Overall Impulse Response:

At E



(b) Integral of Impulse Response:



Problem 4. (15 pts)

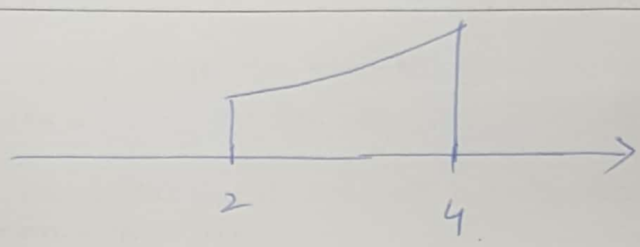
(a) **(10 pts)** Consider an LTI system with impulse response $h(t) = e^t\Big(u(t-2) - u(t-4)\Big)$. If the input to the above system is $x(t) = \Big(u(t+2) - u(t+5)\Big)$, determine the system's output.

(b) **(5 pts)** A cntinuous Time LTI system has input signal $x(t)$ and output signal $y(t)$ related through

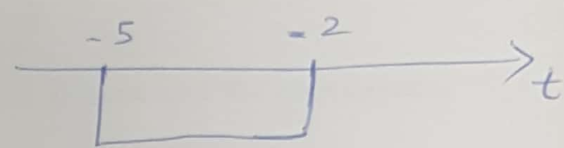
$$y(t) = \int_{t-2}^{t} e^{-(t-\tau)} x(\tau - 2)\, d\tau$$

What is the impulse response for this system? That is, determine $h(t)$ such that $y(t) = x(t) * h(t)$.

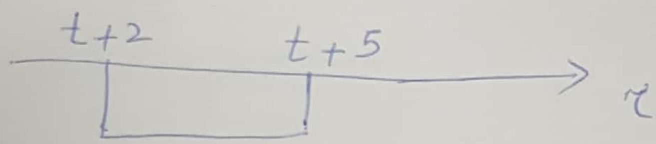
$h(t)$



$x(t)$



$x(t-\tau)$



$t+5 \leq 2$ OR $t+2 \geq 4$ $\equiv$ $t \leq -3$, $t \geq 2$

$y(t) = 0$

$t \geq -3$ AND $t \leq -1$

$$y(t) = \int_{2}^{t+5} -e^{\tau}\, d\tau = e^{2} - e^{t+5}$$

$t \geq -1$ AND $t \leq 0$

$$y(t) = \int_{2}^{4} -e^{\tau}\, d\tau = e^{2} - e^{4}$$

$t \leq 0$ AND $t \leq 2$

$$e^{t+2} - e^{4} = y(t) = \int_{t+2}^{4} -e^{\tau}\, d\tau$$

(b) Change of variable yields

$$h(t) = e^{-(t-2)} \times \left[u(t-2) - u(t-4)\right]$$

Problem 5. (10 pts)

(a) (5 pts) Determine the Fourier series coefficients of the following signal

$$x(t) = \overbrace{2\sum_{n=-\infty}^{\infty}\delta(t-4n)}^{x_1} + \overbrace{\frac{1}{2}\sum_{n=-\infty}^{\infty}(-1)^n\delta(t-2n)}^{x_2}.$$

(b) (5 pts) Consider an LTI system with input $x(t)$ and output $y(t)$ given by

$$x(t) = 15\sin(10\pi t) + 5\cos(30\pi t),$$

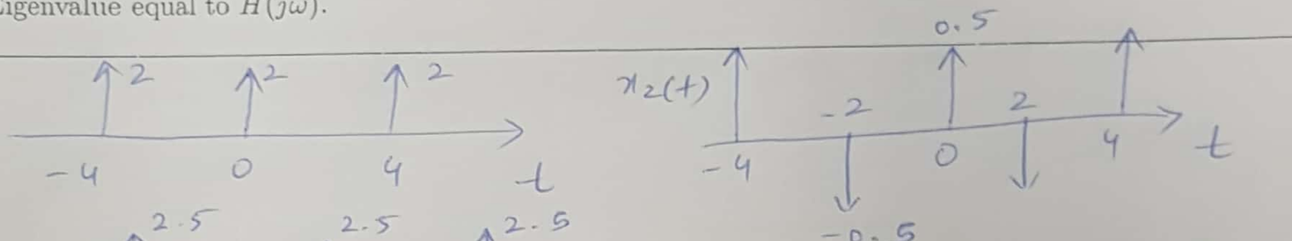
$$y(t) = 20\cos(10\pi t).$$

If $H(jw) \triangleq \int_{\infty}^{\infty} h(t)e^{-jwt}dt$, where $h(t)$ denotes the impulse response of the LTI system, determine $H(j\omega)$ for $\omega = 10\pi,\ -10\pi,\ 30\pi,\ -30\pi$.

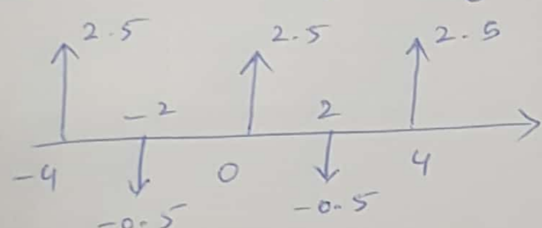
Hint: Use the fact that the signal $e^{j\omega t}$ is an Eigenfunction of LTI systems with the corresponding Eigenvalue equal to $H(j\omega)$.

(a)

$x_1(t)$



$x(t) =$



$T = 4$

$\omega_0 = \pi/2$

$$a_k = 2.5 \times \frac{1}{4} - \frac{0.5}{4} \times e^{-jk\left(\frac{\pi}{2}\right)(2)}.$$

$$\boxed{a_k = \frac{1}{4}\left(2.5 - 0.5(-1)^k\right)}$$

(b) $x(t) = \frac{15}{2j}e^{j10\pi t} - \frac{15}{2j}e^{-j10\pi t} + \frac{5}{2}e^{j30\pi t} + \frac{5}{2}e^{-j30\pi t}$

$y(t) = \frac{20}{2}e^{j10\pi t} + \frac{20}{2}e^{-10j\pi t}$

$\Rightarrow H(j10\pi) = \frac{20}{15}j, \quad H(-j10\pi) = -\frac{20}{15}j$

$H(j30\pi) = H(-j30\pi) = 0.$

Problem 6. **(10 pts)** Consider the following discrete-time periodic signal

$$x[n] = (-1)^n \cos\left(\frac{2\pi}{5}n + \frac{\pi}{3}\right) + e^{j\frac{\pi}{2}n} \sin\left(\frac{4\pi}{5}n + \frac{\pi}{2}\right)$$

Determine (i) a period N for $x[n]$, and (ii) the corresponding Fourier series coefficients $a_0, a_1, \ldots, a_{N-1}$.

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* Noting $(-1)^n = e^{j\pi n}$, Apply Euler angle identities.

$$x[n] = \frac{1}{2} e^{j\frac{7\pi}{5}n} e^{j\pi/3} + \frac{1}{2} e^{j\frac{3\pi}{5}n} e^{-j\pi/3} + \frac{1}{2j} e^{j\frac{13\pi}{10}} e^{j\pi/2} - \frac{1}{2j} e^{-j\frac{3\pi}{10}} e^{-j\pi/2}.$$

$\Rightarrow$ First two terms are periodic with period 10

Last 20

Therefore; $\boxed{N = 20}$, $\boxed{\omega_0 = \frac{\pi}{10}}$

$e^{j\frac{7\pi}{5}} = e^{j14\omega_0}$, $e^{j\frac{3\pi}{5}} = e^{j6\omega_0}$, $e^{j\frac{13\pi}{10}} = e^{j13\omega_0}$, $e^{-j\frac{3\pi}{10}} = e^{-j3\omega_0}$

$\Rightarrow$ Noting $e^{-j\pi/2} = -j$, $e^{j\pi/2} = j$

$\Rightarrow$ $a_{14} = \frac{e^{j\pi/3}}{2}$, $a_6 = \frac{e^{-j\pi/3}}{2}$, $a_{13} = \frac{1}{2}$, $a_{-3} = a_{17} = \frac{1}{2}$

*. Remaining coefficients are zero