Department of Electrical Engineering
School of Science and Engineering

EE310 Signals and Systems

TUTORIAL 7

Information:

- We use the following Fourier transform pairs:

$$\mathcal{F}\{e^{-at}\,u(t)\} = \frac{1}{a+j\omega},$$

$$\mathcal{F}\{u(t)\} = \frac{1}{j\omega} + \pi\delta(\omega).$$

Tutorial 7-1

Using the duality property of Fourier transform, determine the inverse Fourier transform of $X(j\omega) = u(\omega)$.

Tutorial 7-2

The following differential equation relates the output $y(t)$ of causal continuous LTI system to the input $x(t)$

$$\frac{dy(t)}{dt} + 3y(t) \;=\; x(t)$$

(a) Determine the frequency response $H(j\omega) = Y(j\omega)/X(j\omega)$ and sketch the magnitude of $H(j\omega)$.

(b) If $x(t) = e^{-t}u(t)$, determine $Y(j\omega)$ and $y(t)$.

Tutorial 7-3

A casual and stable LTI system has the frequency response

$$H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}$$

and frequency domain description:

$$Y(j\omega) = H(j\omega)\,X(j\omega)$$

(a) Using the property:

$$\mathcal{F}\{\frac{d^k z(t)}{dt^k}\} = (j\omega)^k\,Z(j\omega), \quad k = 0, 1, 2, \ldots$$

where

$$\mathcal{F}\{z(t)\} = Z(j\omega),$$

find the **differential equation** relating the input $x(t)$ (and its derivatives) and output $y(t)$ (and its derivatives) of the system with frequency response $H(j\omega)$.

(b) Find the **partial fraction expansion** of $H(j\omega)$.

(c) Determine the **impulse response** $h(t)$ corresponding to frequency response $H(j\omega)$.