LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE310 Signals and Systems
Quiz 02 - Solutions

Name: ____________________

Campus ID: ____________________

Total Marks: 10
Time Duration: 15 minutes

Question 1 (6 marks)

For a system described by the following input-output relationship:

$$y(t) = \big(2 + \sin(t)\big)\, x(1 - t),$$

where $x(t)$ and $y(t)$ denote the input signal and output signal respectively, determine whether the system is (i) linear, (ii) time-invariant and (iii) causal. *Briefly* justify your answer for each case. No credit will be given for correct answer without justification.

Solution:
(i) System is Linear.
Justification: The system follows the scaling and additivity properties. For inputs $x_1(t)$ and $x_2(t)$, the outputs are $y_1(t) = \big(2 + \sin(t)\big)\, x_1(1 - t)$ and $y_2(t) = \big(2 + \sin(t)\big)\, x_2(1 - t)$ respectively. For input $x_3(t) = \alpha x_1(t) + \beta x_2(t)$, the output is $y_3(t) = \big(2+\sin(t)\big)\, x_3(1-t) = \alpha y_1(t) + \beta y_2(t)$. If we freeze the time at $t = t_o$, the input-output relation is linear, that is, we get a straight line (*passing through origin*) in x-y plane of slope $2\sin(t_o)$.

(ii) System is Time-varying.
Justification: We can show that the output $y_2(t) \neq y_1(t - t_o)$ for input $x_2(t) = x_1(t - t_o)$. Intuitively, the system is not time-invariant due to (i) the factor $2 + \sin(t)$ is not a function of input and (ii) time reversal of the input as the time-shift and reversal (or scaling) do not commute.

(iii) System is Non-causal.
Justification: $y(-3)$ depends on $x(4)$.

Question 2 (4 marks)

For a system described by the following input-output relationship:

$$y[n] = \sum_{k=-3}^{n-4} x[k],$$

where $x[n]$ and $y[n]$ denote the input signal and output signal respectively, determine whether the system is stable and invertible. If the system is invertible, determine the inverse system.

Note that the upper index of summation, in general, cannot be smaller than the lower index. If the upper index is smaller than the lower index, the summation result is zero, that is (for example),

$$\sum_{i=a}^{b} i = 0, \quad \text{for} \quad b < a.$$

Solution:
(i) System is not Invertible.
Justification: It suppresses all the inputs defined for $n < 3$.

(ii) System is not Stable.
Justification: Running sum of the input $x[k]$ from $k = -3$ to $n - 4$. For bounded input $x[n] = u[n]$, the output grows with time.