Quiz 04 Solution

Section 2

Ans 1

(a) $x[n] = 3\cos\left(\frac{\pi}{2}n\right)$

$= 3\left[\frac{1}{2}\left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}\right)\right]$

$x[n] = \frac{3}{2}e^{j\frac{\pi}{2}n} + \frac{3}{2}e^{-j\frac{\pi}{2}n}$

$N = 12 \qquad \omega_0 = \frac{\pi}{6}$

$$x[n] = \frac{3}{2}e^{j\omega_0 n 3} + \frac{3}{2}e^{-j3\omega_0 n}$$

$12\omega_0 = 2\pi \qquad e^{j12\omega_0 n} = e^{j2\pi n} = 1$

but

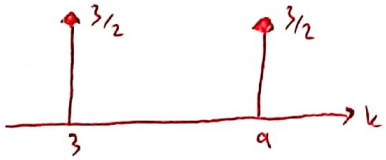
$e^{jn(3\omega_0 + 12\omega_0)} = e^{j9\omega_0 n}$

$\Rightarrow$ compare exponentials with $e^{jk\omega_0 n}$ to find k.

$a_3 = \frac{3}{2}$ so

$a_{-3} = \frac{3}{2}$

$$a_k = \begin{cases} \frac{3}{2}, & k = \pm 3 \text{ or } k = 3, 9 \\ 0, & \text{o.w} \end{cases}$$



$\frac{3}{2}(\delta(k-3) + \delta(k-9))$

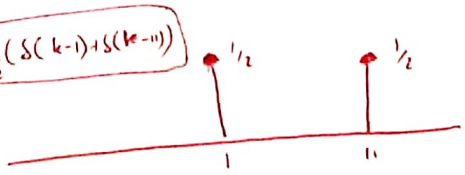
(b) $y[n] = \cos\left(\frac{\pi}{6}n\right)$

$= \frac{1}{2}\left[e^{j\frac{\pi}{6}n} + e^{-j\frac{\pi}{6}n}\right]$

$N = 12 \qquad \omega_0 = \frac{\pi}{6}$

$$y[n] = \frac{1}{2}e^{j\omega_0 n} + \frac{1}{2}e^{-j\omega_0 n}$$

$\frac{1}{2}(\delta(k-1) + \delta(k-11))$



$$b_k = \begin{cases} \frac{1}{2}, & k = \pm 1 \text{ or } k = 1, 11 \\ 0, & \text{o.w} \end{cases}$$

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(c) $z[n] = x[n]\,y[n]$

$$c_k = \sum_{l=\langle N \rangle} a_l\, b_{k-l}$$

for non-zero a_l:

$$c_k = a_3\, b_{k-3} + a_{-9}\, b_{k+9}$$

for non-zero b_k:

$$c_4 = a_3 b_1 = \boxed{\frac{3}{4}} \qquad c_{14} = a_3 b_{11} = \boxed{\frac{3}{4}}$$

$$c_{10} = a_{-9} b_1 = \boxed{\frac{3}{4}} \qquad c_{20} = a_{-9} b_{11} = \boxed{\frac{3}{4}}$$

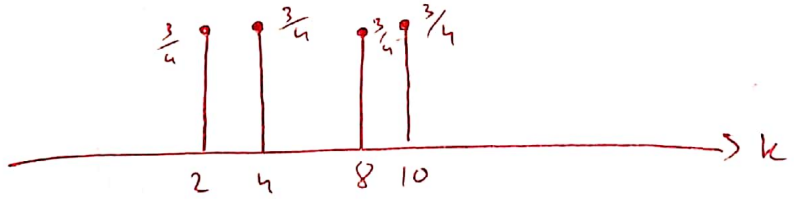
$$c_k = \begin{cases} \frac{3}{4} & ; \quad k = 4, 10, 14, 20 \\ & \quad \text{or} \\ & \quad k = 2, 4, 8, 10 \\ 0 & ; \quad \text{o.w} \end{cases}$$

only need $\boxed{k = 0, \ldots 11}$

But

$\boxed{k=14 \rightarrow k=2}$

$\boxed{k=20 \rightarrow k=8}$



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