LUMS
A Not-for-Profit University

School of Science and Engineering

EE 310 Signals and Systems

PROBLEM SET 1

Problem 1

Evaluate the following expressions involving the Dirac delta function. Carefully justify each result.

1. $\int_{-\infty}^{2} \delta(t-3)\,dt$
2. $\int_{1}^{5} (t-4)\,\delta(t-4)\,dt$
3. $\int_{0}^{\infty} \delta(2t-6)\,dt$
4. $\int_{-2}^{2} \delta(t^2-1)\,dt$

Problem 2

Consider the discrete-time signal $x[n] = 1 - \sum_{k=3}^{\infty} \delta[n-1-k]$.

Determine the values of the integers M and n_0 such that $x[n]$ may be expressed in the form $x[n] = u[Mn - n_0]$.

Problem 3

Determine whether or not each of the following signals is periodic. If the signal is periodic, determine its fundamental period.

1. $x[n] = (-1)^n \cos\left(\frac{2\pi n}{7}\right)$
2. $x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$
3. $x(t) = \sin^2(4t)$
4. $x[n] = \cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right)$

Problem 4

1. Consider a signal $x(t)$ having finite energy E. Answer the following:
   (a) Prove that the signal $x(at)$ has finite energy $\dfrac{E}{|a|}$, where $\forall\, a \in \mathbb{R}$ and $a \neq 0$.
   (b) Now consider the signal $x(t-b)$, where $\forall\, b \in \mathbb{R}$. What will be its energy in terms of E?
   (c) Calculate the energy of the signal $y(-7t+9)$, given that $y(t)$ has finite energy 5. In doing so, you may directly use the results from parts (i) and (ii).
2. Consider the power signal $x(t) = 3 + \frac{1}{2}e^{j\frac{3\pi}{4}t}$. Compute the average power P_∞.

Problem 5

Determine whether each of the following signals is an energy signal, a power signal, or neither. If the signal is an energy or power signal, compute its finite energy or average power.

1. $x(t) = e^{t-\lfloor t \rfloor}$, where $\lfloor t \rfloor$ denotes the greatest integer less than or equal to t (i.e., the floor function).
2. $x(t) = 8\cos\left(\frac{\pi}{2} - 20\pi t\right) + 4\sin(15\pi t)$.
3. $x(t) = e^{-6|t-1|}, \qquad \forall\, t \in \mathbb{R}$.

4. $x(t) = e^{-|t|}\cos(\omega_0 t)$.
5. $x(t) = \sum_{k=-\infty}^{\infty} e^{-|t-k|}$.

Problem 6

For each signal in **Column A**, find the signal or signals in **Column B** that are identical.

Column A	**Column B**
(1) $\delta[n+1]\,u[n]$	(a) $\sum_{k=-\infty}^{\infty} \delta[k]$
(2) $(\frac{1}{2})^n u[n]$	(b) $\frac{d}{dt}u(t)$
(3) $\delta(t)$	(c) $\delta[k]$
(4) $u(t)$	(d) $\sum_{k=0}^{\infty}(\frac{1}{2})^k \delta[n-k]$
(5) $u[n]$	(e) $\int_{-\infty}^{t} \delta(\tau)\,d\tau$
(6) $\delta[n-1]$	(f) $u[n]$
	(g) $\sum_{k=-\infty}^{\infty} (\frac{1}{2})^k \delta[n-k]$
	(h) $\delta[n+1]$
	(i) ϕ

Problem 7

Let $x_1(t) = \sin\big(t + \frac{3\pi}{4}\big)$, $x_2(t) = e^{-\,|t|-2j}$, $x_3[n] = e^{-\,|n-1|}$.

Find the *total energy* and the *average power* of each of the signals $x_1(t)$, $x_2(t)$, and $x_3[n]$.

Problem 8

Let $x_1(t)$ be an energy signal and $x_2(t)$ be a power signal. Let

$$x(t) = x_1(t) + x_2(t).$$

Prove that $x(t)$ is a power signal and not an energy signal.

Problem 9

Prove that for any continuous function $f(t)$:

$$\int_{-\infty}^{\infty} f(t)\,\delta(at-b)\,dt = \frac{1}{|a|} f\Big(\frac{b}{a}\Big), \quad a \neq 0$$

Problem 10

Consider the discrete-time signal defined by a summation of products of unit step functions:

$$x[n] = \sum_{k=0}^{5} u[n-k]\,u[3-k+n],$$

where $u[n]$ is the discrete-time unit step function.

1. Express $x[n]$ as a piecewise function (explicit values for each n).
2. Compute $\sum_{n=-\infty}^{\infty} x[n]$.

Problem 11

Indicate whether each of the following statements is True or False. Justify your answer briefly.

1. The Dirac delta function $\delta(t)$ is a finite-energy signal.
2. The unit step function $u(t)$ is a power signal.
3. Any finite-duration continuous-time signal is always an energy signal.
4. A discrete-time signal $x[n] = e^{j\pi n/3}$ is periodic.
5. A continuous-time signal $x(t) = \sin(\sqrt{2}t)$ is periodic.
6. If $x(t)$ is an energy signal and $y(t)$ is a power signal, $x(t) + y(t)$ is always an energy signal.
7. For a discrete-time signal $x[n]$, if $x[n]$ is periodic, then its energy is infinite.
8. The product of two unit step functions, $u(t-a)u(t-b)$, is equivalent to a single unit step $u(t - \max(a, b))$.

Problem 12

Consider the continuous-time sign function defined as:

$$\text{sgn}(t) = \begin{cases} -1, & t < 0 \\ 0, & t = 0 \\ 1, & t > 0 \end{cases}$$

1. Express $\text{sgn}(t)$ in terms of the unit step function $u(t)$ and finds its derivative.
2. Sketch both $\text{sgn}(t)$ and its derivative.

— End of Problem Set —