LUMS
A Not-for-Profit University

School of Science and Engineering

EE 310 Signals and Systems

PROBLEM SET 2

Problem 1

Express the following numbers in polar form:

(a) $2 + j3$

(b) $-2 + j1$

(c) $-2 - j3$

Problem 2

Represent the following numbers in the complex plane and express them in Cartesian form:

(a) $2e^{j\pi/3}$

(b) $4e^{-j3\pi/4}$

Problem 3

For $z_1 = 2e^{j\pi/4}$ and $z_2 = 8e^{j\pi/3}$, evaluate the following and reflect on whether the phasor (polar/exponential) form or the rectangular (Cartesian) form is more convenient for the operation being performed.

(a) $1/z_1$

(b) z_1/z_2^2

(c) $\sqrt[3]{z_2}$

Problem 4

Express the following in the form $re^{j\theta}$:

(a) $3 + j4$

(b) $\dfrac{6e^{j\pi/3}}{2e^{-j\pi/6}}$

(c) $e^{j\pi/3} \cdot e^{-j\pi/3}$

Problem 5

Express the following in the form $a + jb$:

(a) $5e^{j\pi/6}$

(b) $2e^{-j\pi/3}$

(c) $4e^{j\pi}$

Problem 6

Express each signal using complex exponentials (Euler form):

(a) $x(t) = 4\cos(10t)$

(b) $x(t) = 2\sin(5t)$

(c) $x(t) = 3\cos(8t + \pi/3)$

Problem 7

Write each signal in the form $\Re\{Ce^{j\omega t}\}$ or $\Re\{Ce^{j\omega n}\}$:

(a) $x(t) = 6\cos(4t - \pi/6)$

(b) $x(t) = 2\sin(3t + \pi/3)$

(c) $x[n] = 5\cos(\pi n/5 + \pi/4)$

Problem 8

(a) Determine the even and odd components of the signal

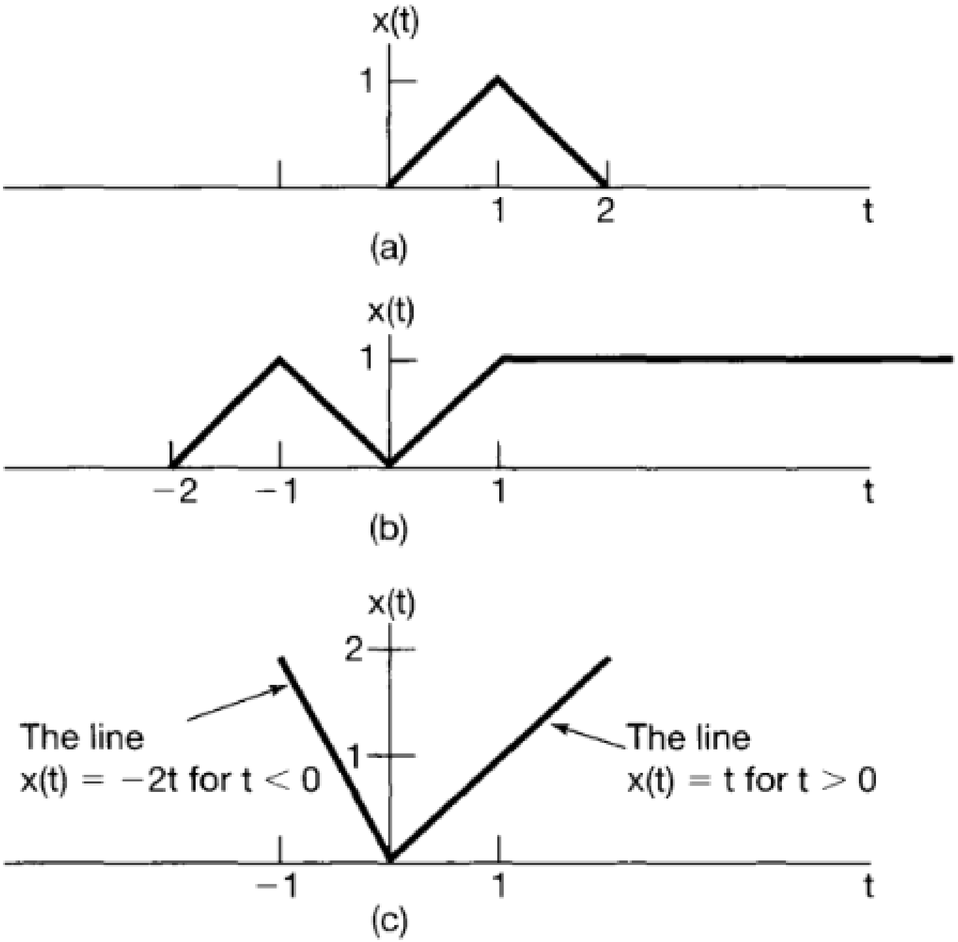
$$x(t) = e^{-2t}u(t).$$

(b) Show that the energy of $x(t)$ is the sum of the energies of its odd and even components found in part (a).

(c) Generalize the result in part (b) for any finite-energy signal.

Problem 9

Determine and sketch the even and odd parts of the signals depicted in the figure below. Label your sketches clearly.



Problem 10

A signal $x[n]$ is defined by the samples:

$$x[-1] = 2, \quad x[0] = 1, \quad x[1] = 0, \quad x[2] = -1, \quad x[3] = 2$$

and $x[n] = 0$ elsewhere.

(a) Sketch $x[n]$.

(b) Find and sketch $x[-n]$.

(c) Find and sketch $x[2-n]$. *Hint: rewrite as* $x[-(n-2)]$.

(d) Find and sketch $y[n] = -x[-n]$.

Problem 11

Let

$$x[n] = 3\cos(0.4\pi n + \pi/6).$$

(a) Write $x[n]$ using Euler's identity as a sum of complex exponentials.

(b) Identify the complex exponential frequencies present.

(c) What is the period of $x[n]$ (if it is periodic)?

(d) Write the corresponding complex exponential form for $3\sin(0.4\pi n + \pi/6)$.

Problem 12

Consider a discrete-time system with input $x[n]$ and output $y[n]$. The input-output relationship for this system is

$$y[n] = x[n]\, x[n-2].$$

(a) Is the system memoryless?

(b) Determine the output of the system when the input is $A\delta[n]$, where A is any real or complex number.

(c) Is the system invertible?

Problem 13

Consider a continuous-time system with input $x(t)$ and output $y(t)$ related by

$$y(t) = x(\sin(t)).$$

(a) Is this system causal?

(b) Is this system linear?

Problem 14

For each of the following input-output relationships, determine whether the corresponding system is linear, time-invariant, or both.

(a) $y(t) = t^2\, x(t-5)$

(b) $y[n] = x^2[n-3]$

(c) $y[n] = x[n+2] - x[n-2]$

Problem 15

You were introduced to a number of general properties of systems. In particular, a system may or may not be: (1) memoryless, (2) time-invariant, (3) linear, (4) causal, (5) stable. Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answers. In each part, $y(t)$ denotes the system output and $x(t)$ is the system input.

(a) $y(t) = x(t-4) + x(4-t)$

(b) $y(t) = \big[\cos 2t\big]\, x(t)$

(c) $y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$

(d) $y(t) = \begin{cases} 0, & t < 0 \\ x(t) + x(t-2), & t \geq 0 \end{cases}$

(e) $y(t) = \dfrac{dx(t)}{dt}$

— End of Problem Set —