LUMS
A Not-for-Profit University

School of Science and Engineering

EE 310 Signals and Systems

PROBLEM SET 2

Problem 1

Express the following numbers in polar form:

(a) $2 + j3$

(b) $-2 + j1$

(c) $-2 - j3$

Solution:

(a) $2 + j3$
$r = \sqrt{2^2 + 3^2} = \sqrt{13}, \quad \theta = \tan^{-1}(3/2)$

$$\boxed{\sqrt{13}e^{j\tan^{-1}(3/2)}}$$

(b) $-2 + j1$
$r = \sqrt{5}, \quad \theta = \pi - \tan^{-1}(1/2)$

$$\boxed{\sqrt{5}e^{j(\pi - \tan^{-1}(1/2))}}$$

(c) $-2 - j3$
$r = \sqrt{13}, \quad \theta = -\pi + \tan^{-1}(3/2)$

$$\boxed{\sqrt{13}e^{j(-\pi + \tan^{-1}(3/2))}}$$

Problem 2

Represent the following numbers in the complex plane and express them in Cartesian form:

(a) $2e^{j\pi/3}$

(b) $4e^{-j3\pi/4}$

Solution:

(a) $2e^{j\pi/3}$

$$= 2(\cos \pi/3 + j \sin \pi/3) = 1 + j\sqrt{3}$$

(b) $4e^{-j3\pi/4}$

$$= 4\left(-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) = -2\sqrt{2} - j2\sqrt{2}$$

$$\boxed{(a)\ 1 + j\sqrt{3} \qquad (b)\ -2\sqrt{2} - j2\sqrt{2}}$$

Problem 3

For $z_1 = 2e^{j\pi/4}$ and $z_2 = 8e^{j\pi/3}$, evaluate the following and reflect on whether the phasor (polar/exponential) form or the rectangular (Cartesian) form is more convenient for the operation being performed.

(a) $1/z_1$

(b) z_1/z_2^2

(c) $\sqrt[3]{z_2}$

Solution:

(a)

$$\frac{1}{z_1} = \frac{1}{2}e^{-j\pi/4}$$

(b)

$$\frac{z_1}{z_2^2} = \frac{2e^{j\pi/4}}{64e^{j2\pi/3}} = \frac{1}{32}e^{-j5\pi/12}$$

(c)

$$\sqrt[3]{z_2} = 2e^{j(\pi/9+2k\pi/3)}, \quad k = 0, 1, 2$$

$$\boxed{\text{Polar form is most convenient for division and roots}}$$

Problem 4

Express the following in the form $re^{j\theta}$:

(a) $3 + j4$

(b) $\dfrac{6e^{j\pi/3}}{2e^{-j\pi/6}}$

(c) $e^{j\pi/3} \cdot e^{-j\pi/3}$

Solution:

(a) $3 + j4$

$$r = 5, \quad \theta = \tan^{-1}(4/3) \Rightarrow \boxed{5e^{j\tan^{-1}(4/3)}}$$

(b)

$$\frac{6e^{j\pi/3}}{2e^{-j\pi/6}} = 3e^{j\pi/2}$$

(c)

$$e^{j\pi/3}e^{-j\pi/3} = e^0 = 1$$

Problem 5

Express the following in the form $a + jb$:

(a) $5e^{j\pi/6}$

(b) $2e^{-j\pi/3}$

(c) $4e^{j\pi}$

Solution:

(a) $5e^{j\pi/6} = \frac{5\sqrt{3}}{2} + j\frac{5}{2}$

(b) $2e^{-j\pi/3} = 1 - j\sqrt{3}$

(c) $4e^{j\pi} = -4$

Problem 6

Express each signal using complex exponentials (Euler form):

(a) $x(t) = 4\cos(10t)$

(b) $x(t) = 2\sin(5t)$

(c) $x(t) = 3\cos(8t + \pi/3)$

Solution:

(a) $4\cos(10t) = 2(e^{j10t} + e^{-j10t})$

(b) $2\sin(5t) = \frac{1}{j}(e^{j5t} - e^{-j5t})$

(c) $3\cos(8t + \pi/3) = \frac{3}{2}(e^{j(8t+\pi/3)} + e^{-j(8t+\pi/3)})$

Problem 7

Write each signal in the form $\Re\{Ce^{j\omega t}\}$ or $\Re\{Ce^{j\omega n}\}$:

(a) $x(t) = 6\cos(4t - \pi/6)$

(b) $x(t) = 2\sin(3t + \pi/3)$

(c) $x[n] = 5\cos(\pi n/5 + \pi/4)$

Solution:

(a) $6\cos(4t - \pi/6) = \Re\{6e^{-j\pi/6}e^{j4t}\}$

(b) $2\sin(3t + \pi/3) = \Re\{2e^{j(\pi/3-\pi/2)}e^{j3t}\}$

(c) $5\cos(\pi n/5 + \pi/4) = \Re\{5e^{j\pi/4}e^{j\pi n/5}\}$

Problem 8

(a) Determine the even and odd components of the signal

$$x(t) = e^{-2t}u(t).$$

(b) Show that the energy of $x(t)$ is the sum of the energies of its odd and even components found in part (a).

(c) Generalize the result in part (b) for any finite-energy signal.

Solution:

(a)

$$x_e(t) = \frac{x(t) + x(-t)}{2}, \quad x_o(t) = \frac{x(t) - x(-t)}{2}$$

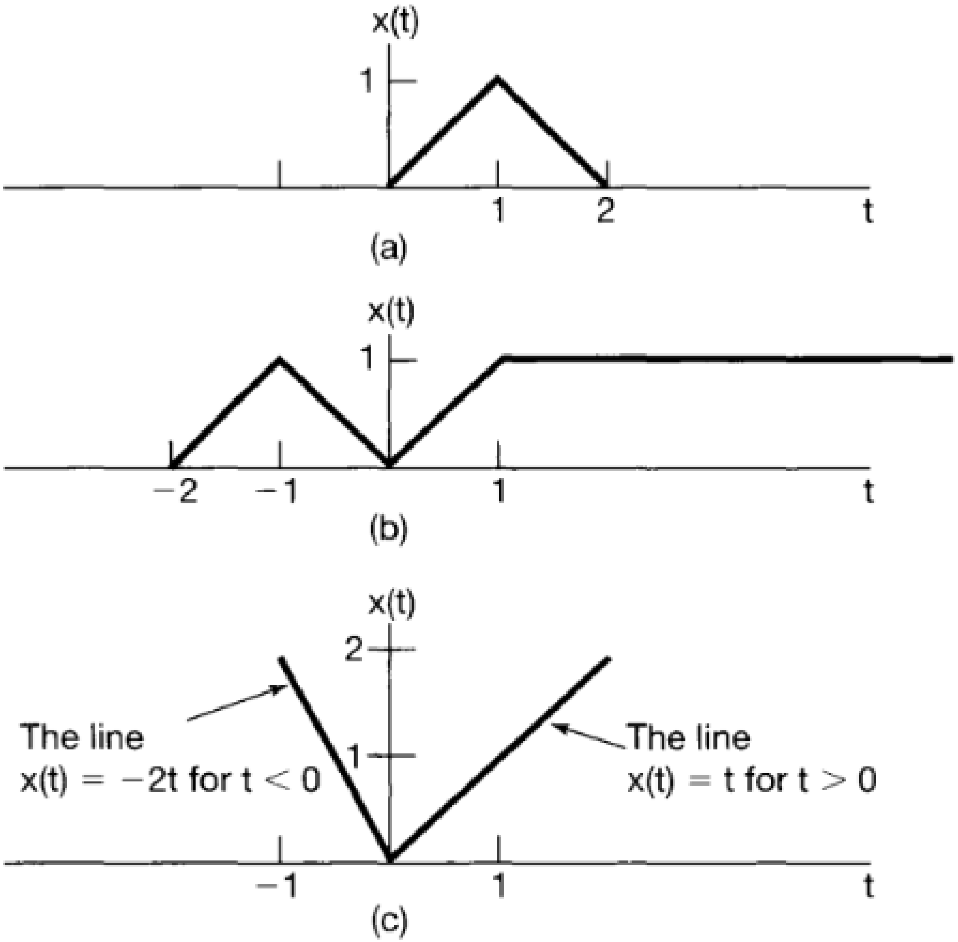
(b)

$$E = E_e + E_o$$

(c) For any finite-energy signal, energy splits orthogonally. Try to connect this concept that the product of even and odd signals will be an odd signal and the integration of the odd signal over time is zero.

Problem 9

Determine and sketch the even and odd parts of the signals depicted in the figure below. Label your sketches clearly.



Solution: Even part is symmetric, odd part is antisymmetric:

$$x_e(t) = \frac{x(t) + x(-t)}{2}, \quad x_o(t) = \frac{x(t) - x(-t)}{2}$$

(Sketch accordingly.)

Problem 10

A signal $x[n]$ is defined by the samples:

$$x[-1] = 2, \quad x[0] = 1, \quad x[1] = 0, \quad x[2] = -1, \quad x[3] = 2$$

and $x[n] = 0$ elsewhere.

(a) Sketch $x[n]$.

(b) Find and sketch $x[-n]$.

(c) Find and sketch $x[2-n]$. *Hint: rewrite as* $x[-(n-2)]$.

(d) Find and sketch $y[n] = -x[-n]$.

Solution:

(a) Sketch given samples.

(b) $x[-n]$ is time-reversed.

(c) $x[2-n]$ is shifted time-reversal.

(d) $y[n] = -x[-n]$ flips amplitude.

Problem 11

Let

$$x[n] = 3\cos(0.4\pi n + \pi/6).$$

(a) Write $x[n]$ using Euler's identity as a sum of complex exponentials.

(b) Identify the complex exponential frequencies present.

(c) What is the period of $x[n]$ (if it is periodic)?

(d) Write the corresponding complex exponential form for $3\sin(0.4\pi n + \pi/6)$.

Solution:

(a)

$$x[n] = \frac{3}{2}\left(e^{j(0.4\pi n+\pi/6)} + e^{-j(0.4\pi n+\pi/6)}\right)$$

(b) Frequencies: $\pm 0.4\pi$

(c) Period: $N = 5$

(d)

$$3\sin(\cdot) = \frac{3}{2j}(e^{j\theta} - e^{-j\theta})$$

Problem 12

Consider a discrete-time system with input $x[n]$ and output $y[n]$. The input-output relationship for this system is

$$y[n] = x[n]\, x[n-2].$$

(a) Is the system memoryless?

(b) Determine the output of the system when the input is $A\delta[n]$, where A is any real or complex number.

(c) Is the system invertible?

Solution:

(a) Not memoryless

(b) $y[n] = 0$

(c) Not invertible

Problem 13

Consider a continuous-time system with input $x(t)$ and output $y(t)$ related by

$$y(t) = x(\sin(t)).$$

(a) Is this system causal?

(b) Is this system linear?

Solution:

(a) Noncausal

(b) Linear

Problem 14

For each of the following input-output relationships, determine whether the corresponding system is linear, time-invariant, or both.

(a) $y(t) = t^2\, x(t-5)$

(b) $y[n] = x^2[n-3]$

(c) $y[n] = x[n+2] - x[n-2]$

Solution:

(a) Linear, not TI

(b) Nonlinear

(c) Linear, not causal

Problem 15

You were introduced to a number of general properties of systems. In particular, a system may or may not be: (1) memoryless, (2) time-invariant, (3) linear, (4) causal, (5) stable. Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answers. In each part, $y(t)$ denotes the system output and $x(t)$ is the system input.

(a) $y(t) = x(t-4) + x(4-t)$

(b) $y(t) = \left[\cos 2t\right] x(t)$

(c) $y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$

(d) $y(t) = \begin{cases} 0, & t < 0 \\ x(t) + x(t-2), & t \geq 0 \end{cases}$

(e) $y(t) = \frac{dx(t)}{dt}$

Solution:

(a) Linear, Stable

(b) Linear, Memoryless, Causal and Stable

(c) Linear

(d) Causal, stable

(e) Linear, causal, Time Invariant

— End of Problem Set —