LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE310 Signals and Systems
Quiz 1 Solutions

Name: ______________________

Campus ID: ______________________

Total Marks: 10
Time Duration: 10 minutes

Question 1 (2 marks)

Determine whether each discrete-time signal is periodic. If periodic, find the *smallest positive period* N_0.

(a) $x_1[n] = \sin\left(\frac{\pi}{5}n\right)$

(b) $x_2[n] = \cos(2n)$

Solution: For a discrete-time sinusoid $x[n] = \cos(\omega_0 n + \phi)$ (or $\sin(\omega_0 n + \phi)$), the signal is periodic iff there exists an integer $N_0 > 0$ such that

$$\omega_0 N_0 = 2\pi k \quad \text{for some } k \in \mathbb{Z}.$$

Equivalently, $\frac{\omega_0}{2\pi}$ must be rational.

(a) For $x_1[n] = \sin\left(\frac{\pi}{5}n\right)$, we need

$$\frac{\pi}{5}N_0 = 2\pi k \quad \Rightarrow \quad N_0 = 10k.$$

The smallest positive period is therefore $N_0 = 10$.

(b) For $x_2[n] = \cos(2n)$, we need

$$2N_0 = 2\pi k \quad \Rightarrow \quad N_0 = \pi k.$$

Since N_0 must be an integer but πk is never an integer for any nonzero integer k, no such N_0 exists. Hence $x_2[n]$ is not periodic.

Question 2 (2 marks)

Evaluate the following:

(a) $\displaystyle\int_{-\infty}^{\infty} (t^2 + 1)\,\delta(t-2)\,dt.$

(b) $\displaystyle\int_{-\infty}^{\infty} \sum_{k=-1}^{1} \delta(k-t)\,dt$

Solution:

(a) $\displaystyle\int_{-\infty}^{\infty} (t^2 + 1)\,\delta(t-2)\,dt = 2^2 + 1 = 5.$

(b) $\displaystyle\int_{-\infty}^{\infty} \sum_{k=-1}^{1} \delta(k-t)\,dt = 3$

Question 3 (3 marks)

Consider the signal

$$x(t) = \begin{cases} t, & 0 \le t \le 1, \\ 1, & 1 < t \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Plot the signal.

(b) Find E_∞ (total energy) and P_∞ (average power).

Solution: $x(t)$ plotted above.

Energy of $x(t)$

$$E_x = \int_0^1 t^2\,dt + \int_1^2 1^2\,dt = \left[\frac{t^3}{3}\right]_0^1 + [t]_1^2 = \frac{1}{3} + 1 = \frac{4}{3}.$$

Since $x(t)$ has finite energy and is time-limited,

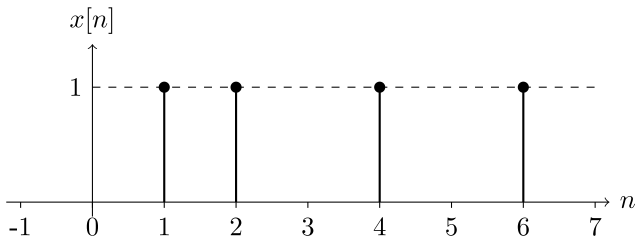
$$P_\infty = 0$$

Question 4 (3 marks)

The discrete-time signal $x[n]$ is shown below.

(a) Express $x[n]$ using a sum of shifted discrete-time impulses $\delta[n-k]$.

(b) Express $x[n]$ using unit steps $u[n]$ (and constants), with no deltas.



Solution: From the plot, $x[n] = 1$ at $n \in \{1, 2, 4, 6\}$ and 0 otherwise.

(a) Impulse (delta) representation:

$$x[n] = \delta[n-1] + \delta[n-2] + \delta[n-4] + \delta[n-6].$$

(b) Step-only representation (using rectangular pulses):

$$x[n] = \big(u[n-1] - u[n-3]\big) + \big(u[n-4] - u[n-5]\big) + \big(u[n-6] - u[n-7]\big).$$