LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE 514 (CS 535) Machine Learning
Quiz 6 Solutions

Name: _____

Campus ID: _____

Total Marks: 10
Time Duration: 15 minutes

Question 1 (6 marks)

Consider a perceptron classifier where the bias term is the first element of the weight vector. The initial weight vector (including bias) is given as:

$$w = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

where the first element represents the bias. Given the following training samples (augmented with a bias input of 1):

$$(x_1, y_1) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad y_1 = 1, \qquad\qquad (x_2, y_2) = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \quad y_2 = -1$$

Apply **one iteration** of the perceptron learning algorithm by updating the weight vector w based on the given samples. Show your calculations.

Solution: The perceptron classification function is:

$$\hat{y} = \text{sign}(w^T x)$$

Step 1: Check the classification of x_1

$$w^T x_1 = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= (0 \times 1) + (1 \times 2) + (-1 \times 3) = 2 - 3 = -1$$

$$\hat{y}_1 = \text{sign}(-1) = -1$$

Since $\hat{y}_1 \neq y_1$ (misclassification), we update the weight vector:

$$w \leftarrow w + y_1 x_1$$

$$w = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

Step 2: Check the classification of x_2

$$w^T x_2 = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

$$= (1 \times 1) + (3 \times 1) + (2 \times -2) = 1 + 3 - 4 = 0$$

$$\hat{y}_2 = \text{sign}(0) = 0$$

Since $\hat{y}_2 \neq y_2$ (misclassification), we update the weight vector:

$$w \leftarrow w + y_2 x_2$$

$$w = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}$$

Final Updated Weights after one iteration:

$$\boxed{w = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}}$$

Question 2 (4 marks)

In the logistic regression model, suppose you have learned the following parameters from the data:

$$\theta_0 = -3, \quad \theta_1 = 1, \quad \theta_2 = 2$$

Write the equation of the decision boundary in terms of $x^{(1)}$ and $x^{(2)}$ and also draw the boundary in 2-dimensional plane. Classify the data point $x = [1, 2]$.

Solution: The decision boundary is given by $\theta^T x = 0$, that is

$$-3 + x^{(1)} + 2x^{(2)} = 0$$

$$x^{(1)} + 2x^{(2)} = 3$$

In $x^{(1)} - x^{(2)}$ plane, the boundary will be a line of slope $-\frac{1}{2}$ and $x^{(2)}$ intercept of $\frac{3}{2}$.

For the data point $x = [1, 2]$:

$$\theta^T x = -3 + 1 \cdot 1 + 2 \cdot 2 = -3 + 1 + 4 = 2 > 0$$

Since $\theta^T x > 0$, the data point is classified as Class 1.