LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE 514 (CS 535) Machine Learning
Quiz 7 Solutions

Name: ______________________

Campus ID: ______________________

Total Marks: 10
Time Duration: 15 minutes

Question 1 (4 marks)

(a) [3 marks] Mathematically formulate how C appears in the primal optimization problem of soft-margin SVM and explain its role in the objective function. Analyze the effect of varying C on the width of the margin.

Solution: The primal optimization problem with parameter C:

$$\min_{\mathbf{w},b,\xi} \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{n}\xi_i$$

subject to:

$$y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1 - \xi_i \quad \text{and} \quad \xi_i \geq 0 \quad \forall i$$

Here, C controls the tradeoff between 'margin maximization ($\|\mathbf{w}\|^2$ minimization)' and 'penalty for violations ($\sum \xi_i$)'.
Effect of varying C:

- **Large** C:
  - Narrower margin (larger $\|\mathbf{w}\|$)
  - Fewer classification errors (strict enforcement)
  - Potentially fewer support vectors
  - Risk of overfitting
- **Small** C:
  - Wider margin (smaller $\|\mathbf{w}\|$)
  - More tolerance for errors
  - Typically more support vectors
  - Risk of underfitting

(b) [1 mark] In the dual SVM formulation, the Lagrange multipliers α_i are non-zero only for:

A) All training points
B) Misclassified points
C) Support vectors
D) Outliers

Solution: C) Support vectors

Question 2 (6 marks)

Given the following linearly separable dataset:

- **Class +1 (Positive):** $(1, 1)$ and $(1, 3)$
- **Class -1 (Negative):** $(3, 1)$ and $(3, 3)$

(a) Plot the data and identify the support vectors.

(b) Find the optimal hyperplane (decision boundary) using Hard SVM.

(c) Determine the weight vector $\mathbf{w}$ and bias term θ.

(d) Compute the classification margin ρ

Solution: We need to:

$$\text{minimize } \frac{1}{2}\|\mathbf{w}\|^2$$

subject to:

$$\begin{aligned} w_1 + w_2 - \theta &\geq 1 \quad \text{(for (1,1))} \\ w_1 + 3w_2 - \theta &\geq 1 \quad \text{(for (1,3))} \\ 3w_1 + w_2 - \theta &\leq -1 \quad \text{(for (3,1))} \\ 3w_1 + 3w_2 - \theta &\leq -1 \quad \text{(for (3,3))} \end{aligned}$$

By symmetry, we can assume $w_2 = 0$. Then:

$$\begin{aligned} w_1 - \theta &= 1 \\ 3w_1 - \theta &= -1 \end{aligned}$$

Solving gives:

$$w_1 = -1, \theta = -2$$

Thus:

$$\mathbf{w} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \theta = -2$$

The decision boundary is:

$$-x + 2 = 0 \quad \text{or} \quad x = 2$$

Margin boundaries:

$$\begin{aligned} -x + 2 = 1 &\Rightarrow x = 1 \\ -x + 2 = -1 &\Rightarrow x = 3 \end{aligned}$$

Margin width:

$$\rho = \frac{2}{\|\mathbf{w}\|} = \frac{2}{1} = 2$$