Lahore University of Management Sciences

EE 518 –Data Analysis and AI on the Sphere

Fall 2022

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Office Hours	
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Course URL (if any)	Current:

Course Teaching Methodology (Please mention following details in plain text)
• Teaching Methodology: In-Person • Attendance is not mandatory but maintaining a good record will help students in many ways. Students not frequently attending the lecture will find difficult to cope with the course. We may take attendance during the session and monitor your presence in the class.

Course Basics				
Credit Hours	3			
Lecture(s)	Nbr of Lec(s) Per Week	2	Duration	1 hour and 15 minutes
Tutorial (per week)	Nbr of Lec(s) Per Week	0/1	Duration	1 hour (tentative)

Course Distribution	
Core	
Elective	This is an elective graduate level course.
Open for Student Category	Open for Graduates, Seniors and Juniors
Close for Student Category	

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COURSE DESCRIPTION
Data defined on the 2-dimensional surface of the 2-sphere arise whenever the underlying configuration of the problem has a spherical geometry. Hence, such data has inherent angular dependence and is encountered in various fields of science and engineering, such as wireless communication, 3D beamforming, computer graphics, medical imaging, acoustics, geodesy, quantum chemistry, cosmology, astronomy and planetary sciences. Examples of data on the sphere include gravitational, topographic and magnetic fields of the planets, cosmic microwave background (CMB), a relic radiation from the Big Bang, pouring down on us from space, electromagnetic radiation pattern of an antenna array in spherical geometry, acoustic signal received using spherical configuration of microphones etc. As opposed to 1D and multidimensional Euclidean domain data, which are defined on flat Euclidean domains, spherical data is defined on a curved domain due to which extension of Euclidean domain analysis and processing techniques to the spherical domain is not a trivial task. However, over the years, a lot of methods developed for Euclidean domain signals/data have been carefully extended to process data on the sphere to serve the needs of above-mentioned applications and scientific areas. This course provides a thorough introduction to the theoretical foundations of data analysis, processing and learning on the sphere. Furthermore, the students will also learn to visualize the data in the spatial (spherical) and harmonic domain using Matlab.

COURSE PREREQUISITE(S)	
•	Pre-requisites: None
•	Co-requisites: None
	The students are expected to have a basic understanding of linear algebra, vector spaces, Hilbert spaces and functional analysis

COURSE OBJECTIVES	
	In broad brush, this course aims to: • Provide a thorough introduction to both fundamental and advanced topics related to data representation on the sphere in both the spatial and harmonic domains. • Provide understanding of Hilbert space of spherical signals and different subspaces • understand the notion of rotation of the data and its formulation in both the spatial and spectral domain. • Provide a detailed review of Slepian spatial-spectral concentration problem and its applications. • Develop computationally efficient methods to analyze and process data on the sphere. • Train the students to Implement convolution neural networks on the sphere and carry out so-called geometric deep learning. • Apply these data analysis methods in 2-3 application areas.

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Learning Outcomes	
	The students should be able to:
CLO1:	Understand the core theoretical concepts serve as basis for data representation on the sphere
CLO2:	Understand the spectral domain representation of the data using spherical harmonics
CLO3:	Understand the data acquisition methods, sampling strategies, spherical harmonic transform, rotation, convolutional neural networks and geometric deep learning
CLO4:	Implement the algorithms for data analysis on the sphere using MATLAB

Grading break up: Component Details and weightages

Assignments, 40 %
Quizzes, 20 %
Mid-Exam, 20 %
Final Exam, 20 %

Examination Detail	
Midterm Exam	Yes/No: Yes Combine Separate: Combined Duration: 120 minutes Preferred Date: TBA Exam Specifications: TBA
Final Exam	Yes/No: Yes Combine Separate: Combined Duration: 180 minutes Exam Specifications: TBA

Textbook(s)/Supplementary Readings

- R.A. Kennedy Hilbert Space Methods in Signal Processing, Cambridge, March 2013.
- Research Articles
- Class notes will be provided to supplement these readings

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Outline of Topics and Tentative Weekly Schedule	
Module	Topic
Fundamentals, Spherical Harmonics (2-3 weeks)	Course Overview, Motivation: Applications of data analysis on the sphere
	Advanced operations on vectors, norm, angle, inner product
	2-sphere domain, Inner product , Hilbert space
	Spherical harmonics, spectral domain characterization
	Subspaces: Space-limited, band-limited, azimuthally symmetric and order-limited subspaces
Rotations and Convolution (2 weeks)	Rotations on the sphere
	SO(3) representation
	Harmonic domain characterization of rotation
	SO(3) rotation group – data representation
	Wigner-D functions
	Convolution on the Sphere
Sampling and Data Acquisition (2 weeks)	Sampling of data on the sphere and SO(3)
	Notion of optimal dimensionality
	Equiangular sampling, Gauss-Legendre sampling, Optimal dimensionality sampling
Slepian Spatial-Spectral Concentration Problem (3 weeks)	Slepian concentration problem on the sphere
	Band-limited eigenfunctions with optimal spatial domain concentration
	Space-limited eigenfunctions with optimal harmonic domain concentration
	Equivalence of problems
	Slepian Basis - Properties and significance for data representation
	Computation of Slepian functions for different spatial regions
	Application: Multi-taper spectrum estimation on the sphere
Joint-domain transforms (2 weeks)	Spatial-Spectral and Spatial-Slepian transforms
	Joint domain representations
	Filtering and estimation in the joint domains
Stochastic Processes on Sphere (1 week)	Anisotropic processes, isotropic processes, azimuthally symmetric processes Spatial statistics Application to climate modelling
Spherical CNNs (2 weeks)	Convolution formulation on the sphere
	Architecture of convolutional neural network and learning framework