**LAHORE UNIVERSITY OF MANAGEMENT SCIENCES**
**Department of Electrical Engineering**

**EE563 Convex Optimization**
**Spring 2020**

**Assignment-4**

Posted on: April 26, 2020
Submission Deadline: 23:55 pm, Wednesday, May 6, 2020
**NOTE:** Submit your assignment on LMS. Go to Assignments tab on the LMS page where you can find the dropbox for assignment submission. You are encouraged to submit the assignment in the form of a single pdf file.

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**Problem 1, Textbook 4.23)** (10 points)
Formulate the $l_4$-norm optimization problem

$$\text{minimize } ||Ax - b||_4 = (\textstyle\sum_{i=1}^{n}(a_i^T x - b_i)^4)^{1/4}$$

as a QCQP for given matrix $A \in \mathbf{R}^{m\times n}$ (with rows $a_i^T$) and the vector $b \in \mathbf{R}^m$.

**Problem 2)** (10 points)
We consider a following optimization problem

$$\begin{array}{ll} \text{maximize} & f_0(x) \\ \text{subject to} & f_i(x) \leq 0, \quad i = 1, \ldots, m \\ & x \succeq 0, \end{array}$$

where each $f_i$ is quadratic, that is, $f_i(x) = (1/2)x^T P_i x + q_i^T x + r_i$, with $P_i \in \mathbf{S}^n$, $q_i \in \mathbf{R}^n$ and $r_i \in \mathbf{R}$ for $i = 0, \ldots, m$. We note here that the problem may not be convex since we have not restricted $P_i$ to be positive semi-definite.

Suppose that $q_i \preceq 0$ and $P_i$ is a $Z$-matrix for $i = 0, \ldots, m$. We refer to a matrix with nonpositive off-diagonal elements as $Z$-matrix.) Explain how to reformulate this problem as a convex problem.

*Hint.* Use change of variable approach to obtain an equivalent problem, that is, use $y_j = \phi(x_j)$ by choosing a suitable function $\phi$. Use the information that $x$ is non-negative in every component to determine $\phi$ (scalar function, $\mathbf{R}_+ \to \mathbf{R}_+$).

**Problem 3)** (10 points)
Consider a following optimization problem

$$\text{maximize} \quad \Big(\prod_{i=1}^{m}(a_i^T x - b_i)\Big)^{1/m},$$

where we have an implicit constraint that the domain of the problem is $\{x \in \mathbf{R}^n \| a_i T x \geq b_i, \ , i = 1, 2, \ldots, m\}$. Formulate the given optimization problem as SOCP.

**Problem 4, Textbook 4.40)** (15 points)
Express LP, QP, QCQP and SOCP as SDP.

**Problem 5)** (25 points)
Formulate each of the the following optimization problems as a semidefinite program. For a variable $x \in \mathbf{R}^n$, $F(x)$ is defined as as

$$F(x) = F_0 + x_1 F_1 + x_2 F_2 + \cdots + x_n F_n,$$

where $F_i \in \mathbf{S}^m$..

a) Minimize $\lambda_1(x)$, where $\lambda_1(x)$ represents the largest eigenvalue of the matrix $F(x)$.

b) Minimize $f(x) = c^T F(x)^{-1} c$ where $c \in \mathbf{R}^m$, with $\mathbf{dom} f = \{x \in \mathbf{R}^n | F(x) \succeq 0\}$.

c) Minimize $f(x) = \max\limits_{i=1,\ldots,K} c_i^T F(x)^{-1} c_i$ where $c_i \in \mathbf{R}^m$, $i = 1, \ldots, K$, with $\mathbf{dom} f = \{x \in \mathbf{R}^n | F(x) \succeq 0\}$.

d) Minimize $f(x) = \sup\limits_{||c||_2 \leq 1} c^T F(x)^{-1} c$, with $\mathbf{dom} f = \{x \in \mathbf{R}^n | F(x) \succeq 0\}$.

e) Minimize $f(x) = \mathbf{E}(c^T F(x)^{-1} c)$ where c is a random vector with mean $\mathbf{E} c = \bar{c}$ and covariance $\mathbf{E}(c - \bar{c})(c - \bar{c})^T = S$ and $\mathbf{dom} f = \{x \in \mathbf{R}^n | F(x) \succeq 0\}$.

**Problem 6)** (30 points) We consider a power assignment problem in a wireless communication system.

a) Formulate the following power assignment problem as a generalized linear-fractional program (defined in Section 4.3.2).

**System Model:** Consider a wireless communication system with $n$ transmitters transmitting to $n$ receivers. The problem under consideration is to determine the power to be transmitted by each transmitter. We have the following information:

- $0 \leq p_i \leq P_i^{\max}$ denotes the power transmitted by $i$-th transmitter, where $P_i^{\max} > 0$ is the maximum power available at $i$-th transmitter.
- The transmitters are divided into $m$ number of groups $K_1, K_2, \ldots, K_m$ such that $K_1 \cup K_2 \cup \ldots \cup K_m = \{1, 2, \ldots, n\}$ and $K_p \cap K_q = \emptyset$ for $p \neq q$. The total transmitter power cannot exceed $P_\ell^{\text{gp}}$ for each group, that is,

$$\sum_{i \in K_\ell} p_i = P_\ell^{\text{gp}}, \quad \ell = 1, 2, \ldots, m.$$

- $G_{ij} \geq 0$ denotes the path gain from transmitter $j$ to receiver $i$.
- The total power $y_i$ received by the $i$-th receiver is given by

$$y_i = \underbrace{G_{ii} p_i}_{S_i,\ \text{Signal Power}} + \underbrace{\sum_{j=1, j \neq i}^{n} G_{ij} p_j}_{I_i,\ \text{Interference Power}} + \underbrace{\sigma_i}_{\text{Noise Power}}$$

- The total power received at the $i$-th receiver from all the transmitters cannot exceed $P_i^{\text{rc}}$.

**Power Assignment Problem:** We want to determine the power to be transmitted by the transmitters such that the constraints are satisfied and minimum Signal to Interference plus Noise Ratio (SINR) is maximized. Here SINR for the $i$-th receiver is given by $S_i/(I_i + \sigma_i)$ and minimum is taken over the number of receivers.

b) Solve the maximization of minimum SINR formulated in the previous part using CVX. Use the following data and information.

- $n = 5$ transmitters partitioned into two groups: $K_1 = \{1, 2\}$ and $K_2 = \{3, 4, 5\}$
- $P_i^{\max} = 5\,\text{Watts (W)}$ for $i = 1, 2, \ldots, 5$
- $P_1^{\text{gp}} = 4\,\text{W}$ and $P_2^{\text{gp}} = 6\,\text{W}$
- Noise power $\sigma_i = 0.5\,\text{W}$ for $i = 1, 2, \ldots, 5$
- $P_i^{\text{rc}} = 5\,\text{W}$ for $i = 1, 2, \ldots, 5$
- The path gains are defined by the following matrix

$$G = \begin{bmatrix} 1 & 0.1 & 0.2 & 0.1 & 0 \\ 0.1 & 1 & 0.1 & 0.1 & 0 \\ 0.2 & 0.1 & 2 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.2 & 1.0 & 0.1 \\ 0 & 0.0 & 0.2 & 0.1 & 1 \end{bmatrix}$$

- Since you will be implementing a bi-section algorithm (pseudo-code given in Section 4.2.5), you will be required to check the feasibility of the problems. You can use the following command `strcmpi(cvx_status, 'Solved')` to check the feasibility of the problem.