LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE563 Convex Optimization
Spring 2020

Assignment-5

Posted on: Friday, May 8 , 2020
Submission Deadline: 23:55, Friday, May 15, 2020
NOTE: Submit your assignment on LMS. Go to Assignments tab on the LMS page where you can find the dropbox for assignment submission. You are encouraged to submit the assignment in the form of a single pdf file.

Problem 1) (10 points)
Consider an optimization problem (primal)

$$\begin{array}{ll} \text{minimize} & f_o(x) = \sum_{k=1}^{n} x_k \log(x_k/y_k) \\ \text{subject to} & Ax = b \\ & \mathbf{1}^T x = 1 \end{array}$$

We note that the objective function in this problem given by as

$$f_o(x) = \sum_{k=1}^{n} x_k \log(x_k/y_k),$$

represents the relative entropy between two vectors $x, y \in \mathbf{R}^n_{++}$ and is a convex function jointly in x and y. The optimization variable is $x \in \mathbf{R}^n$ and we assume $x \succ 0$, that is, we consider $\mathbf{R}^n_{++}$ as domain of the objective function. In the problem above, we compute the vector x that minimizes the relative entropy with a given vector $y \succ 0$ subject to equality constraints on x.

Show that the Lagrange dual problem can be formulated as

$$\text{maximize} \quad b^T z - \log \textstyle\sum_{k=1}^{n} y_k e^{a_k^T z},$$

where a_k represents the kth column of A and z is the Lagrange dual variable associated with the equality constraint $Ax = b$.

Problem 2) (06+06 points)
Consider a source localization problem. In wireless communication, we can estimate the distance of the transmitter using the signal strength of the received signal since the signal is expected to decrease (according to power law) with the distance. This information is used in wireless communication to locate a user. Your mobile phone signal (usually) communicates with multiple base stations at the same time.

Consider a problem with m number of sensors (receivers) at known positions $y_1, \ldots, y_m \in \mathbf{R}^n$ ($n = 2$ or $n = 3$) and 1 transmitter at an unknown position $x \in \mathbf{R}^n$ ($n = 2$ or $n = 3$). From the strength of the signals received by the sensors from the transmitter, we can obtain noisy estimates d_k of the distances $\|x - y_k\|_2$. Utilizing these distances, we are interested in estimating the source position x by solving the following optimization problem that minimizes the error between the squares of the observed and actual distances, i.e.,

$$\text{minimize} \quad f_0(x) = \sum_{k=1}^{m} \left(\|x - y_k\|_2^2 - d_k^2\right)^2$$

.

By using a new scalar variable $t = x^T x$, we can reformulate the problem as

$$\begin{array}{ll} \text{minimize} & \sum_{k=1}^{m} \left(t - 2y_k^T x + \|y_k\|_2^2 - d_k^2\right)^2 \\ \text{subject to} & x^T x - t = 0. \end{array} \tag{1}$$

We note that the problem is not convex. However, we assume that the strong duality holds.

For $m = 5$ and $n = 2$, we consider the following data to solve a problem.

$$y_1 = \begin{bmatrix} 1.8 \\ 2.5 \end{bmatrix}, \quad y_2 = \begin{bmatrix} 2.0 \\ 1.7 \end{bmatrix}, \quad y_3 = \begin{bmatrix} 1.5 \\ 1.5 \end{bmatrix}, \quad y_4 = \begin{bmatrix} 1.5 \\ 2.0 \end{bmatrix}, \quad y_5 = \begin{bmatrix} 2.5 \\ 1.5 \end{bmatrix},$$

and

$$d = (2.00, 1.24, 0.59, 1.31, 1.44).$$

a) We consider an optimization variable $z = (x_1, x_2, t)$. If the optimization problem given in (1) can be formulated as

$$\begin{array}{ll} \text{minimize} & \|Az - b\|_2^2 \\ \text{subject to} & z^T C z + 2f^T z = 0, \end{array} \tag{2}$$

where

$$b = \begin{bmatrix} d_1^2 - \|y_1\|_2^2 \\ d_2^2 - \|y_2\|_2^2 \\ d_3^2 - \|y_3\|_2^2 \\ d_4^2 - \|y_4\|_2^2 \\ d_5^2 - \|y_5\|_2^2 \end{bmatrix},$$

determine A, C and f.

b) Derive a Lagrange dual problem for the optimization problem given in (2).

c) Formulate KKT conditions for the optimization problem given in (2).

d) Express the Lagrange dual problem as SDP and solve it using CVX. Use the data provided to determine optimal dual variable μ^*.

e) Using the KKT condition(s) and optimal dual variable computed in the previous part, determine optimal variable z^*.