EE563 Convex Optimization

Assignment 05 Solution

Problem 1 The Lagrangian is

$$L(x, z, \mu) = \sum_k x_k \log(x_k/y_k) + b^T z - z^T A x + \mu - \mu \mathbf{1}^T x.$$

Minimizing over x_k gives the conditions

$$1 + \log(x_k/y_k) - a_k^T z - \mu = 0, \quad k = 1, \ldots, n,$$

with solution

$$x_k = y_k e^{a_k^T z + \mu - 1}.$$

Plugging this in in L gives the Lagrange dual function

$$g(z, \mu) = b^T z + \mu - \sum_{k=1}^{n} y_k e^{a_k^T z + \mu - 1}$$

and the dual problem

$$\text{maximize} \quad b^T z + \mu - \textstyle\sum_{k=1}^n y_k e^{a_k^T z + \mu - 1}.$$

This can be simplified a bit if we optimize over μ by setting the derivative equal to zero:

$$\mu = 1 - \log \sum_{k=1}^{n} y_k e^{a_k^T z}.$$

Substituting μ in Lagrange dual problem yields

$$\text{maximize} \quad b^T z - \log \textstyle\sum_{k=1}^n y_k e^{a_k^T z}$$

Problem 2

a)

$$A = \begin{bmatrix} -2y_1^T & 1 \\ -2y_2^T & 1 \\ \vdots & \vdots \\ -2y_5^T & 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad f = \begin{bmatrix} 0 \\ 0 \\ -1/2 \end{bmatrix}$$

b) The Lagrangian is

$$L(z,\mu) = z^T(A^TA + \mu C)z - 2(A^Tb - \mu f)^T z + \|b\|_2^2,$$

which is bounded below as a function of z only if

$$A^TA + \mu C \succeq 0, \qquad A^Tb - \mu f \in \mathcal{R}(A^TA + \mu C).$$

The KKT conditions are therefore as follows.

- *Primal feasibility.*
$$z^TCz + 2f^Tz = 0.$$
- *Dual feasibility.*
$$A^TA + \mu C \succeq 0, \qquad A^Tb - \mu f \in \mathcal{R}(A^TA + \mu C).$$
- *Gradient of Lagrangian is zero.*
$$(A^TA + \mu C)z = A^Tb - \mu f.$$

c) We derive the dual problem. If μ is feasible, then

$$g(\mu) = -(A^Tb - \mu f)^T(A^TA + \mu C)^\dagger(A^Tb - \mu f) + \|b\|_2^2,$$

so the dual problem can be expressed as an SDP

$$\begin{array}{ll} \text{maximize} & -t + \|b\|_2^2 \\ \text{subject to} & \begin{bmatrix} A^TA + \mu C & A^Tb - \mu f \\ (A^Tb - \mu f)^T & t \end{bmatrix} \succeq 0. \end{array}$$

Solving this in CVX gives $\mu^\star = 0.5896$.

d) Using μ^* in the stationarity condition, we obtain

$$z^\star = (A^TA + \mu C)^{-1}(A^Tb - \mu f) = (1.33,\ 0.64,\ 2.18).$$

Hence $x^\star = (1.33, 0.64)$.