LUMS
A Not-for-Profit University

Department of Electrical Engineering
School of Science and Engineering

EE563 Convex Optimization

TUTORIAL 2

Textbook: Convex Optimization – Boyd and Vandenberghe

Tutorial 2-1

Show that the log-sum-exponential function $f : \mathbf{R}^n \to \mathbf{R}$ given by

$$f(x) = \log \sum_{i=1}^{n} e^{x_i}$$

is convex on $\mathbf{R}^n$.

Tutorial 2-2

Determine whether the following functions are convex, quasi-convex, concave or quasi-concave.

(a) $f(x_1, x_2) = x_1 x_2, \quad \mathbf{dom} f = R^2_{++}$

(b) $f(x_1, x_2) = \frac{x_1}{x_2}, \quad \mathbf{dom} f = R^2_{++}$

(c) $f(x_1, x_2) = \frac{\|Ax+b\|_2^2}{c^T x+d}, \quad \mathbf{dom} f = \{x | cTx + d > 0\}$

Tutorial 2-3

The induced norm of a matrix $X \in \mathbf{R}^{p \times q}$ is given by

$$\|X\|_{a,b} = \sup_{v \neq 0} \frac{\|Xv\|_a}{\|v\|_b}.$$

Show that the induced norm is a convex function.

Tutorial 2-4

Determine the conjugate function of the following function

$$f(X) = -\log \det X, \quad \mathbf{dom} f = S^n_{++}.$$

Tutorial 2-5

The maximum generalized eigenvalue of symmetric matrices $X \in \mathbf{S}^n$ and $Y \in \mathbf{S}^n_{++}$ is defined as

$$\lambda_{\max}(X, Y) = \sup_{v \neq 0} \frac{v^T X v}{v^T Y v}.$$

Determine whether the function $\lambda_{\max} : \mathbf{S}^n \times \mathbf{S}^n_{++} \to \mathbf{R}$ is convex, quasi-convex, concave or quasi-concave.

Tutorial 2-6

Problem 3.24

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