LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE563/MATH325 Convex Optimization (Spring 2020)
Quiz 01

Name: _____

Campus ID: _____

Total Marks: 10
Time Duration: 20 minutes

Question 1 (4 marks)

Show that the set $S \subseteq \mathbf{R}^3$ given by

$$S = \big\{ x \in \mathbf{R}^n \big| \, -2 \leq \text{Real}\big(\sum_{k=1}^{n} x_k e^{jkt} \big) \leq 4 \quad \text{for} \quad |t| \leq 3 \big\}$$

is convex. Here $j = \sqrt{-1}$ denotes the complex imaginary number, $e^{jkt} = \cos(kt) + j\sin(kt)$ and $\text{Real}(\cdot)$ returns the real part of the complex argument.

Question 2 (3 marks)

Consider a proper cone $K = \{x \in \mathbf{R}^2 | x_i \leq 0, i = 1, 2\}$.

(a) [1 mark] Sketch the cone K.

(b) [2 marks] Find the minimum (if any) or minimal element(s) of the set $S = R_+^2 \cap B$ with respect to the cone K, where $B = \{x \in \mathbf{R}^2 | \|x\|_2 \leq b\}$ is the Euclidean norm ball in $\mathbf{R}^2$.

Question 3 (3 marks)

(a) [1 mark] Define the dual of the cone K, that is, provide an explicit definition of the dual cone.

(b) [2 marks] Find the dual of the cone $K = \{x \in \mathbf{R}^2 | x_1 \leq -|x_2|\}$.