LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE563/MATH325 Convex Optimization (Spring 2020)
Quiz 01 - Solutions

Name: _____

Campus ID: _____

Total Marks: 10
Time Duration: 20 minutes

Question 1 (4 marks)

Show that the set $S \subseteq \mathbf{R}^n$ given by

$$S = \big\{ x \in \mathbf{R}^n \big| -2 \leq \text{Real}\big(\sum_{k=1}^{n} x_k e^{jkt}\big) \leq 4 \quad \text{for} \quad |t| \leq 3 \big\}$$

is convex. Here $j = \sqrt{-1}$ denotes the complex imaginary number, $e^{jkt} = \cos(kt) + j\sin(kt)$ and $\text{Real}(\cdot)$ returns the real part of the complex argument.

Solution: For each t, the set S is given by

$$S_t = \big\{ x \in \mathbf{R}^n \big| -2 \leq c^T x \leq 4 \big\},$$

where $c = \big(\cos(t), \cos(2t), \ldots, \cos(nt)\big)$. S_t is an intersection of two half-spaces and is therefore convex. S is convex since it is an intersection of convex sets S_t, that is $S = \bigcap_{|t| \leq 3} S_t$.

Question 2 (3 marks)

Consider a proper cone $K = \{x \in \mathbf{R}^2 | x_i \leq 0, i = 1, 2\}$.

(a) [1 mark] Sketch the cone K.

Solution: The cone is non-positive orthant.

(b) [2 marks] Find the minimum (if any) or minimal element(s) of the set $S = R_+^2 \cap B$ with respect to the cone K, where $B = \{x \in \mathbf{R}^2 | \|x\|_2 \leq b\}$ is the Euclidean norm ball in $\mathbf{R}^2$.

Solution: There is no minimum element. $\{x \in \mathbf{R}_+^2 | \|x\|_2 = b\}$ represents a set of minimal elements.

Question 3 (3 marks)

(a) [1 mark] Define the dual of the cone K, that is, provide an explicit definition of the dual cone.

Solution: The dual cone K^* is defined as

$$K^* = \{x | x^T y \geq 0, \forall\, y \in K\}.$$

(b) [2 marks] Find the dual of the cone $K = \{x \in \mathbf{R}^2 | x_1 \leq -|x_2|\}$.

Solution: The cone of self-dual.