

LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE563/MATH325 Convex Optimization (Spring 2020)
Quiz 02

Name: _____

Campus ID: _____

Total Marks: 20

Time Duration: 25 minutes

Question 1 (8 marks)

Determine whether each of the following functions defined on \mathbf{R}^n or specified domain is convex or concave or neither. Provide brief justification to support your answer.

(a) $f(x) = a^T x + b, \quad x \in \mathbf{R}^n$

(b) $f(x) = \frac{1}{1-x^T x}, \quad \|x\|_2 < 1, \quad x \in \mathbf{R}^n$

(c) $f(x) = e^x + e^{-x}, \quad x \in \mathbf{R}$

(d) $f(x) = \max\{1/x, x^2, e^x\}, \quad x \in \mathbf{R}_+$

(e) $f(x, y, z) = -\log(yz - x^T x), \quad \text{dom} f = \{x \in \mathbf{R}^n, y, z \in \mathbf{R}_{++} \mid yz > x^T x\}$

Question 2 (3 marks)

Consider a polynomial

$$P(x, \omega) = x_1 + x_2 \cos \omega + x_3 \cos 2\omega + \cdots + x_n \cos(n-1)\omega.$$

Show that the function

$$f(x) = -\int_0^{2\pi} \log P(x, \omega) d\omega,$$

is convex on $\{x \in \mathbf{R}^n \mid P(x, \omega) > 0, \omega \in [0, 2\pi]\}$.

Question 3 (4 marks)

If $f : \mathbf{R}^n \rightarrow \mathbf{R}$ is a convex function show that its α -sub-level set S_α given by

$$S_\alpha = \{x \in \mathbf{R}^n \mid f(x) \leq \alpha\}$$

is convex.

Question 4 (5 marks)

Determine the conjugate of the function $f(x) = x^p, \quad x \in \mathbf{R}_{++}, \quad p > 1$.