LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE563/MATH325 Convex Optimization (Spring 2020)
Quiz 03 - Solutions

Total Marks: 15
Time Duration: 30 minutes

Question 1 (8 marks)

We have two optimization problems:

Problem 1:

$$\begin{aligned} \text{minimize} \quad & a^T x \\ \text{subject to} \quad & x \succeq 0 \\ & Gx = b \end{aligned}$$

Problem 2:

$$\begin{aligned} \text{minimize} \quad & (a^T x)^2 \\ \text{subject to} \quad & x \succeq 0 \\ & Gx = b \end{aligned}$$

(a) [3 marks] Do these problems always have the same solution? Provide example(s) to show that these problem do not necessarily share the same solution.

Solution: We just take a scalar case, i.e., $x \in \mathbf{R}$. If you choose $a = -3$, we have the following optimal solution and optimal value:

Problem 1: $x^* = \infty$, $p^* = -\infty$ (unbounded below) Problem 2: $x^* = 0$, $p^* = 0$

(b) [5 marks] Formulate the Problem 2 as a linear program (optimization problem).

Solution: Linear program with variables x and t (scalar):

$$\begin{aligned} \text{minimize} \quad & t \\ \text{subject to} \quad & x \succeq 0 \\ & a^T x \leq t \\ & t \geq 0 \\ & Gx = b \end{aligned}$$

Question 2 (7 marks)

A manager of an oil refinery plant has 2 million barrels of crude oil A and 4 million barrels of crude oil B allocated for production during the coming month. Refinery can use these resources to produce either gasoline (sale price \$38 per barrel) or home heating oil (sale price \$33 per barrel). For the production of these fuels, refinery has three production processes with the following characteristics:

All quantities are in barrels. For example, with the first process, 3 barrels of crude A and 5 barrels of crude B are used to produce 4 barrels of gasoline and 3 barrels of heating oil. The costs in this table refer to variable and allocated overhead costs and there are no separate cost items for the

Table 1: Characteristics of the Production processes

	Process 1	Process 2	Process 3
Input crude A	3	1	5
Input crude B	5	1	3
Output Gasoline	4	1	3
Output Heating Oil	3	1	4
Cost	\$51	\$11	\$40

cost of the crudes. Formulate an optimization problem that would help the manager to determine the optimal production level, that is, how much each of the processes to be utilized, such that the net revenue over the next month is maximized.

Solution: Choose x, y, z as variables for the production processes 1, 2 and 3 respectively and formulate a linear program with objective function (revenue) given by

$$f_o(x, y, z) = 38(4x + y + 3z) + 33(3x + y + 4z) - (51x + 11y + 40z) = 200x + 60y + 206z,$$

and inequality constraints given by

$$3x + y + 5z \leq 2,000,000$$
$$5x + y + 3z \leq 4,000,000$$
$$[x, y, z]^T \succeq 0$$