Total Marks: 10
Time Duration: 30 minutes

Question 1 (10 marks)

Consider a following optimization problem

$$\text{minimize} \quad \max_{k=1,2,\ldots,p} |\log(a_k^T x) - \log(b_k)|$$
$$\text{subject to} \quad x \succeq 0,$$

where we assume that $b_i > 0$ and $\log(a_i^T x) = -\infty$ when $a_i^T x \leq 0$ for $i = 0, 1, \ldots, n$. Formulate the problem as SOCP.

You may find the following relationships/information useful.

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$$|\log p - \log q| = \log \ \max(p/q, q/p), \quad p, q \geq 0.$$

- log is monotone increasing and therefore minimizing the log of any function is equivalent to the log of the minimum of the function.

- Hyperbolic constraint $w^T w \leq yz$ for $y, z \geq 0$ can be expressed as a second-order constraint given by

$$\left\| \begin{bmatrix} 2w \\ y - z \end{bmatrix} \right\|_2 \leq y + z.$$