LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE563/MATH325 Convex Optimization (Spring 2020)
Quiz 04

Total Marks: 10
Time Duration: 30 minutes

Question 1 (10 marks)

Consider a following optimization problem

$$\begin{array}{ll} \text{minimize} & \max\limits_{k=1,2,\dots,p} |\log(a_k^T x) - \log(b_k)| \\ \text{subject to} & x \succeq 0, \end{array}$$

where we assume that $b_i > 0$ and $\log(a_i^T x) = -\infty$ when $a_i^T x \leq 0$ for $i = 0, 1, \dots, n$. Formulate the problem as SOCP.

Solution: Noting the following

$$|\log(a_k^T x) - \log(b_k)| = \log\ \max(\frac{a_k^T x}{b_k}, \frac{b_k}{a_k^T x}),$$

we can rewrite the above problem as

$$\begin{array}{ll} \text{minimize} & \max\limits_{k=1,2,\dots,p} |\log(a_k^T x) - \log(b_k)| = \max\limits_{k=1,2,\dots,p} \log\ \max(\frac{a_k^T x}{b_k}, \frac{b_k}{a_k^T x}) \\ \text{minimize} & \max\limits_{k=1,2,\dots,p} \max(\frac{a_k^T x}{b_k}, \frac{b_k}{a_k^T x}), \end{array}$$

since log is monotone increasing; minimizing the log of any function is equivalent to the log of the minimum of the function.

Now using the epigraph reformulation, we can write the optimization problem as

$$\begin{array}{ll} \text{minimize} & t, \\ \text{subject to} & \frac{a_k^T x}{b_k} \leq t, \quad k = 1, 2, \dots, p \\ & \frac{a_k^T x}{b_k} \geq \frac{1}{t}, \quad k = 1, 2, \dots, p \\ & x \succeq 0. \end{array}$$

By reformulating the constraint $\frac{a_k^T x}{b_k} \geq \frac{1}{t}$ as

$$\left\| \begin{bmatrix} 2 \\ t - \frac{a_k^T x}{b_k} \end{bmatrix} \right\|_2 \leq t + \frac{a_k^T x}{b_k}, \quad k = 1, 2, \dots, p,$$

we obtain SOCP in variables x and t. □

You may find the following relationships/information useful.

- $$|\log p - \log q| = \log\ \max(p/q, q/p), \quad p, q \geq 0.$$
- log is monotone increasing and therefore minimizing the log of any function is equivalent to the log of the minimum of the function.

- Hyperbolic constraint $w^T w \leq yz$ for $y, z \geq 0$ can be expressed as a second-order constraint given by

$$\left\| \begin{bmatrix} 2w \\ y - z \end{bmatrix} \right\|_2 \leq y + z.$$