

Convex Optimization

Machine Learning Application: Linear Classifier

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Outline

- Classification Problem (binary)
- Linear Classifier Design as Optimization Problem

Classification Problem in Machine Learning

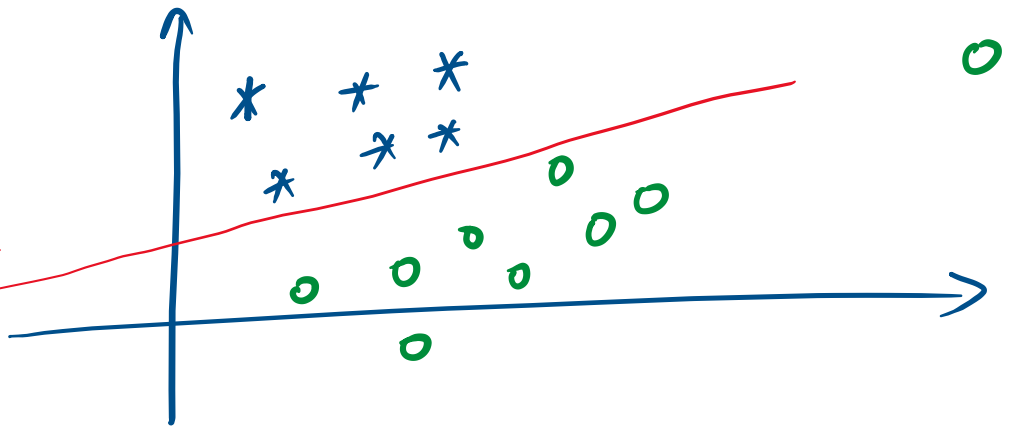
- * Data points for 2 classes (Label)
- * Design a classifier for classifying new data point.

- *

class 0	class 1
$p_1, p_2, \dots, p_p \in \mathbb{R}^n$	$q_1, q_2, \dots, q_q \in \mathbb{R}^n$

$w \in \mathbb{R}^n$
 $b \in \mathbb{R}$

$w^T x = b$
 This is Hyperplane



Classification Problem in Machine Learning

Given $\underbrace{p_1, p_2, \dots, p_p}_{\text{Class 0}}, \underbrace{q_1, q_2, \dots, q_q}_{\text{Class 1}}$

Determine $w, b \longrightarrow w^T x \neq b$
 $\downarrow \quad \downarrow$
 $\mathbb{R}^n \quad b$

such that $w^T p_i > b \quad i = 1, \dots, p$
 $w^T q_i < b \quad i = 1, \dots, q$

Classifier
Problem

How to formulate this design as an optimization problem?

Classification Problem in Machine Learning

Solution 01

$$\begin{array}{ll} \text{minimize} & 100 \\ \text{subject to} & w^T p_i \geq b \quad i = 1, \dots, P \\ & w^T q_j \leq b \quad j = 1, \dots, Q \end{array}$$

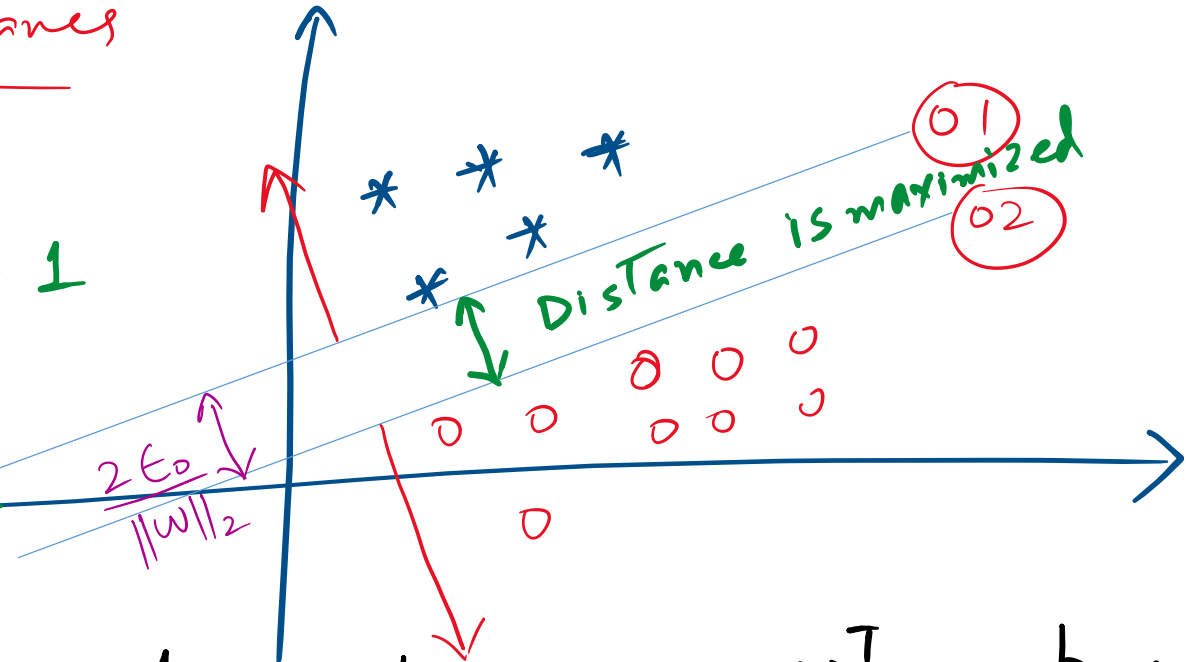
* $w = 0, b = 0$ (solution)

Classification Problem in Machine Learning

Two parallel Hyperplanes

- (01) $w^T x = b + \epsilon_0$
 (02) $w^T x = b - \epsilon_0$
- $\epsilon_0 = 1$

Parallel Hyperplanes



Distance b/w Parallel Hyperplanes

$$w^T x = b + \epsilon_0$$

* w ; normal

$$\Delta = \frac{b + \epsilon_0}{\|w\|_2} - \frac{b - \epsilon_0}{\|w\|_2} = \frac{2\epsilon_0}{\|w\|_2}$$

* $\frac{b + \epsilon_0}{\|w\|_2}$ (Distance of plane from origin)

Classification Problem in Machine Learning

$$\text{maximize}_{w \in \mathbb{R}^n, b \in \mathbb{R}} \frac{2\epsilon_0}{\|w\|_2} \quad \text{minimize } \|w\|_2$$

$$\text{Subject to} \quad \begin{aligned} &\rightarrow w^T p_i \geq b + \epsilon_0 \quad i = 1, \dots, P \\ &\rightarrow w^T q_j \leq b - \epsilon_0 \quad j = 1, \dots, Q \end{aligned}$$

Convex Optimization Problem

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Support Vector Machine	SVM
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Feedback: Questions or Comments?

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Slides available at: https://www.zubairkhalid.org/ee563_2020.html

(Let me know should you need latex source)