Convex Optimization

Duality: Geometric Interpretation

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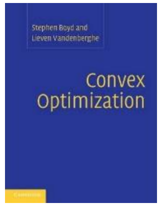
Outline

- Duality Geometric Interpretation



Section 5.3

Duality

Primal Optimization problem

$$\begin{aligned} \text{minimize} \quad & f_o(x) \\ \text{subject to} \quad & f_i(x) \leq 0, \quad i = 1, 2, \ldots, m \\ & h_j(x) = 0, \quad j = 1, 2, \ldots, p \end{aligned}$$

Lagrange Dual Problem:

$$\begin{aligned} \text{maximize} \quad & g(\lambda, \mu) \\ \text{subject to} \quad & \lambda \succeq 0 \end{aligned}$$

- Optimal points: x^* and (λ^*, μ^*)

Duality – Geometric Interpretation

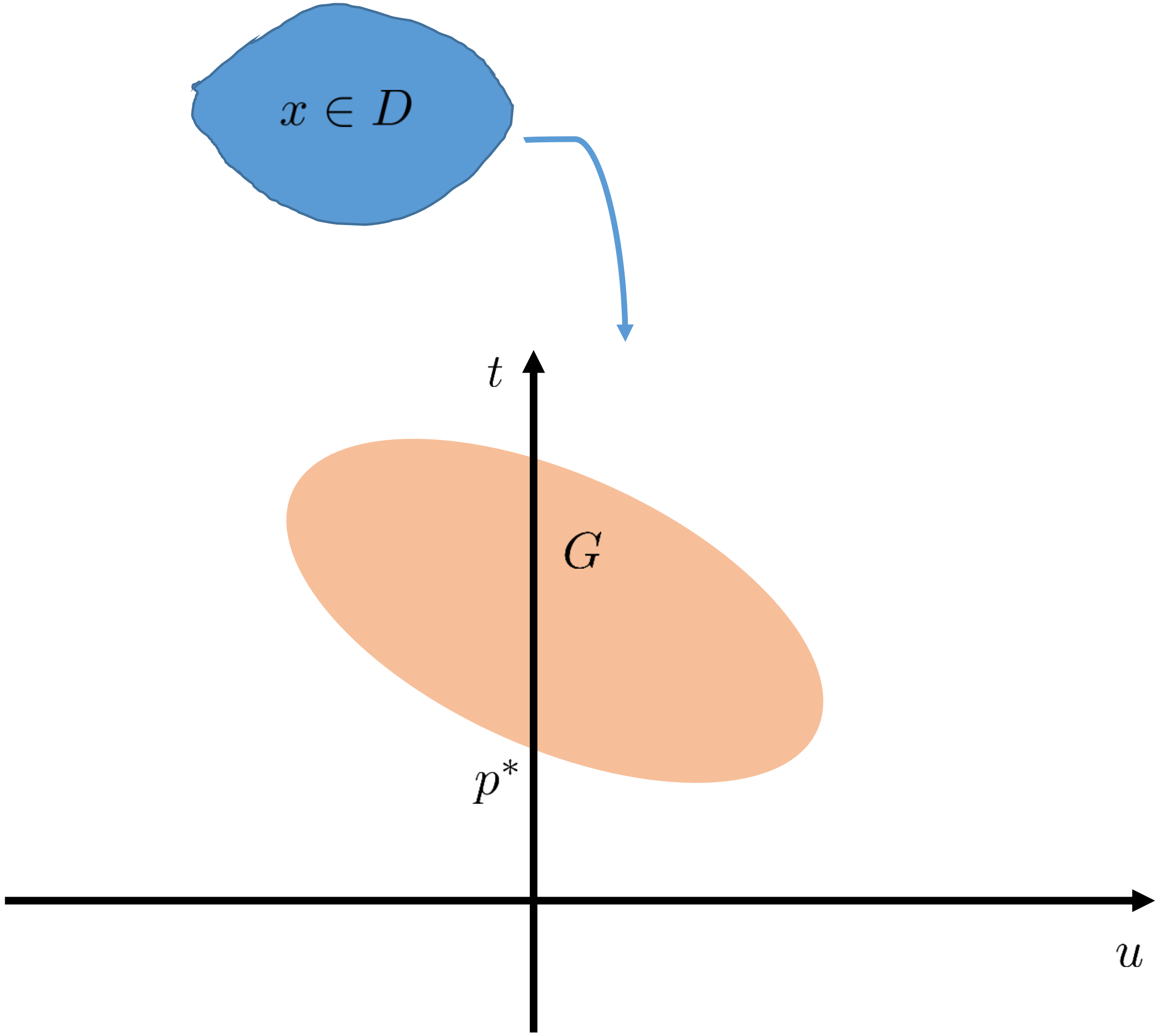
Primal Optimization problem

minimize $f_o(x)$

subject to $f_1(x) \leq 0$

$$G = \{(u,t) | f_o(x) = t,\ f_1(x) = u,\ x \in D\}$$

D - domain of the problem



Duality – Geometric Interpretation

Lagrange Dual Function:

$$g(\lambda) = \inf_{x \in D} f_o(x) + \lambda f_1(x)$$

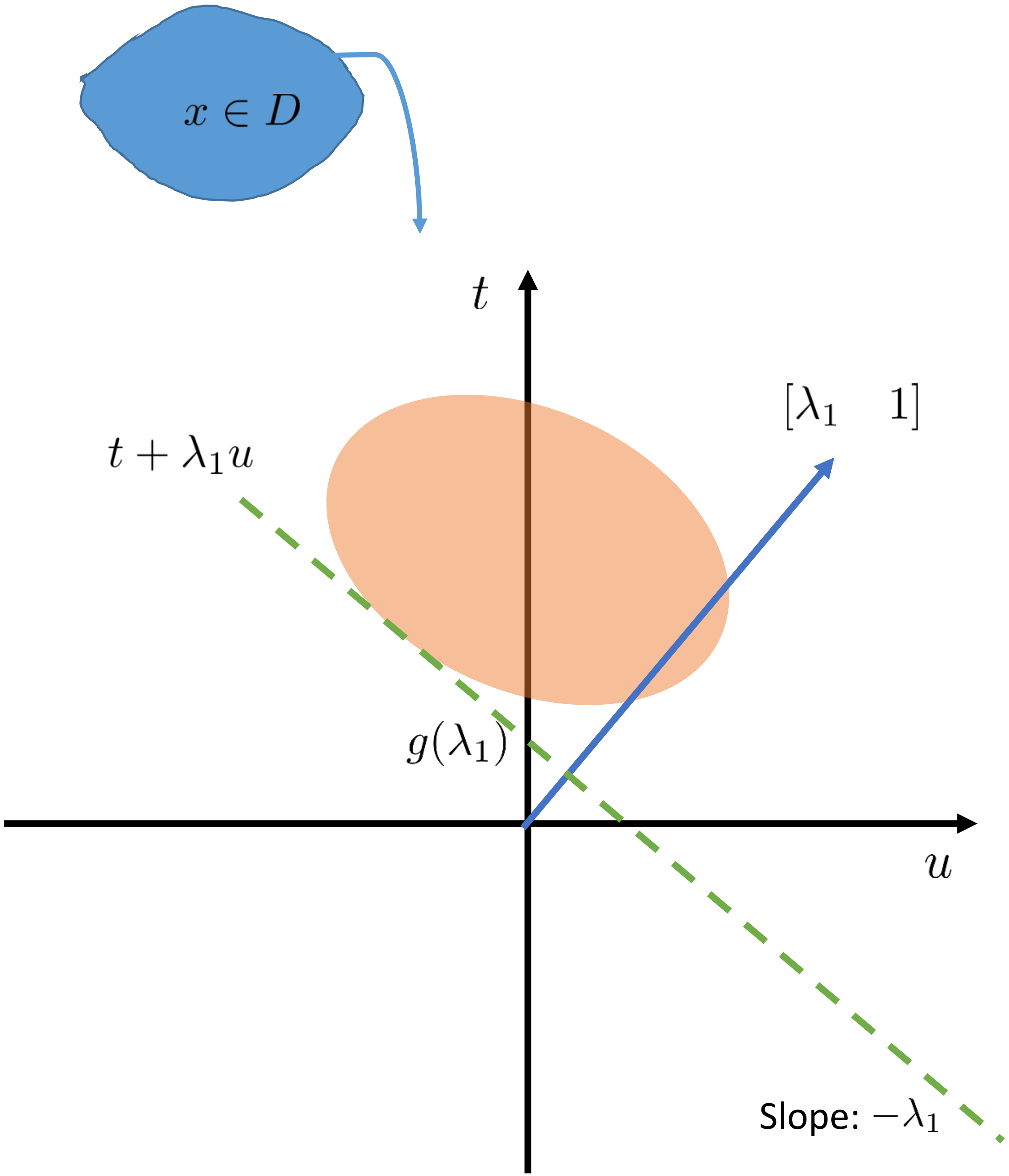
Noting:

$$G = \{(u,t) | f_o(x) = t,\ f_1(x) = u,\ x \in D\}$$

We re-write:

$$g(\lambda) = \inf_{(u,t) \in G} t + \lambda u$$

$$g(\lambda) = \inf_{(u,t) \in G} [\lambda \quad 1][u \quad t]^T$$



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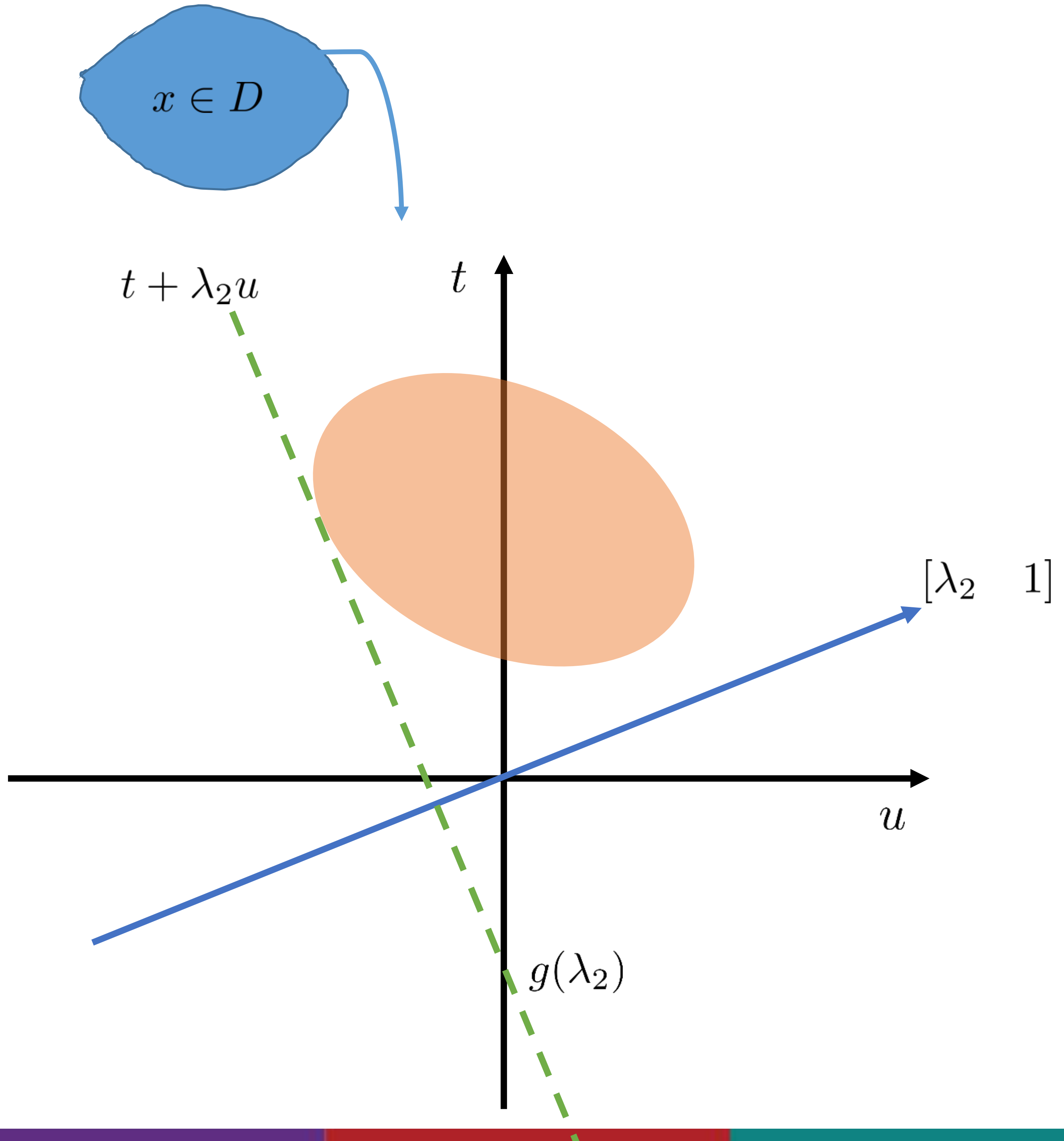

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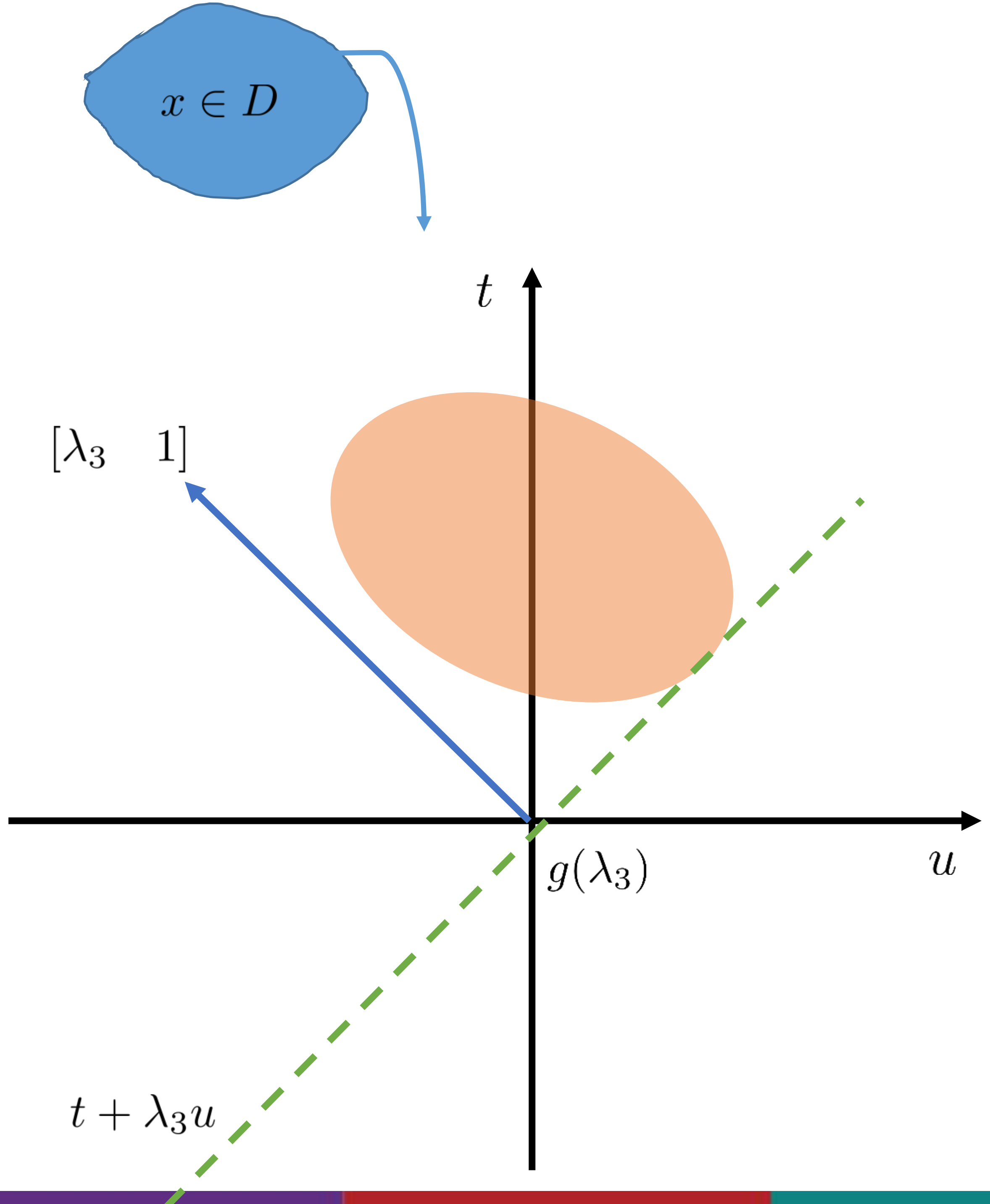

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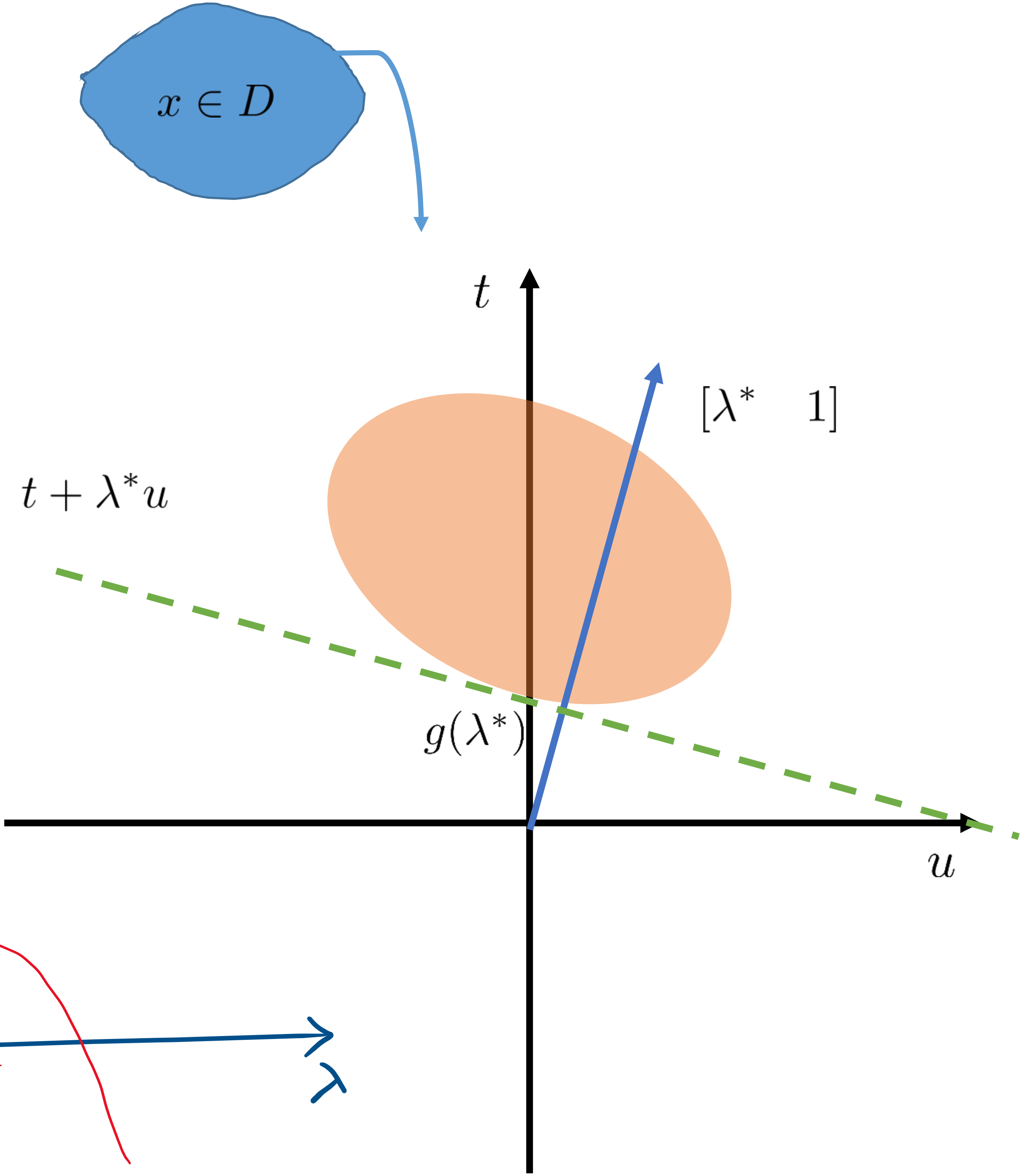

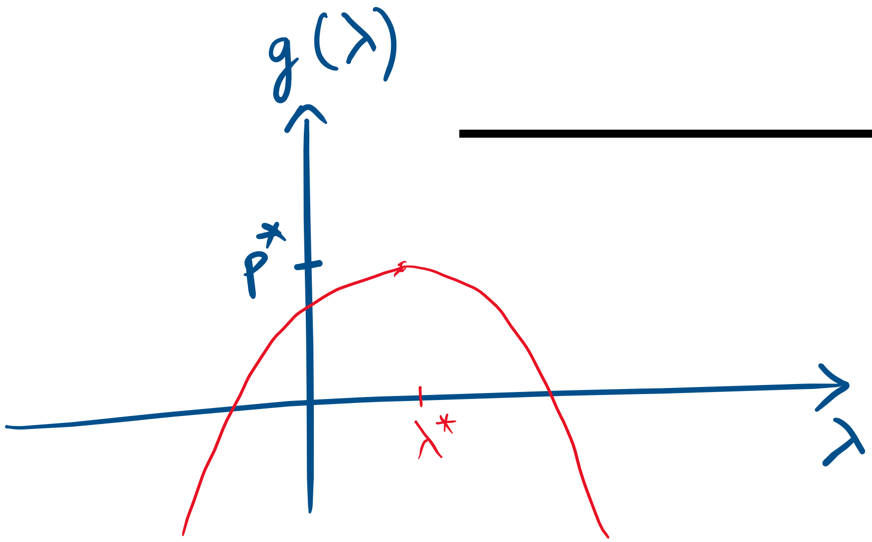

Duality – Geometric Interpretation (Weak Duality)

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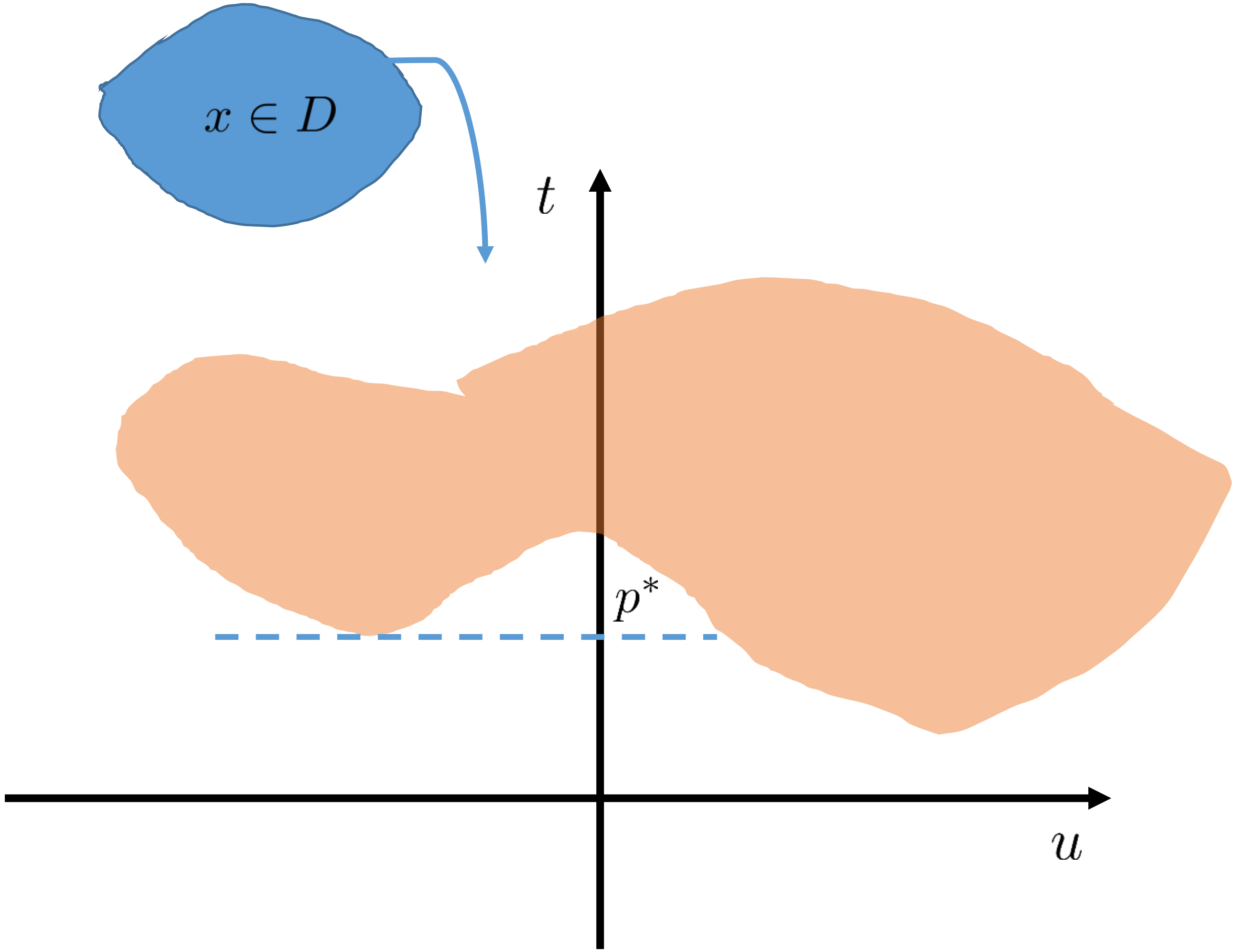

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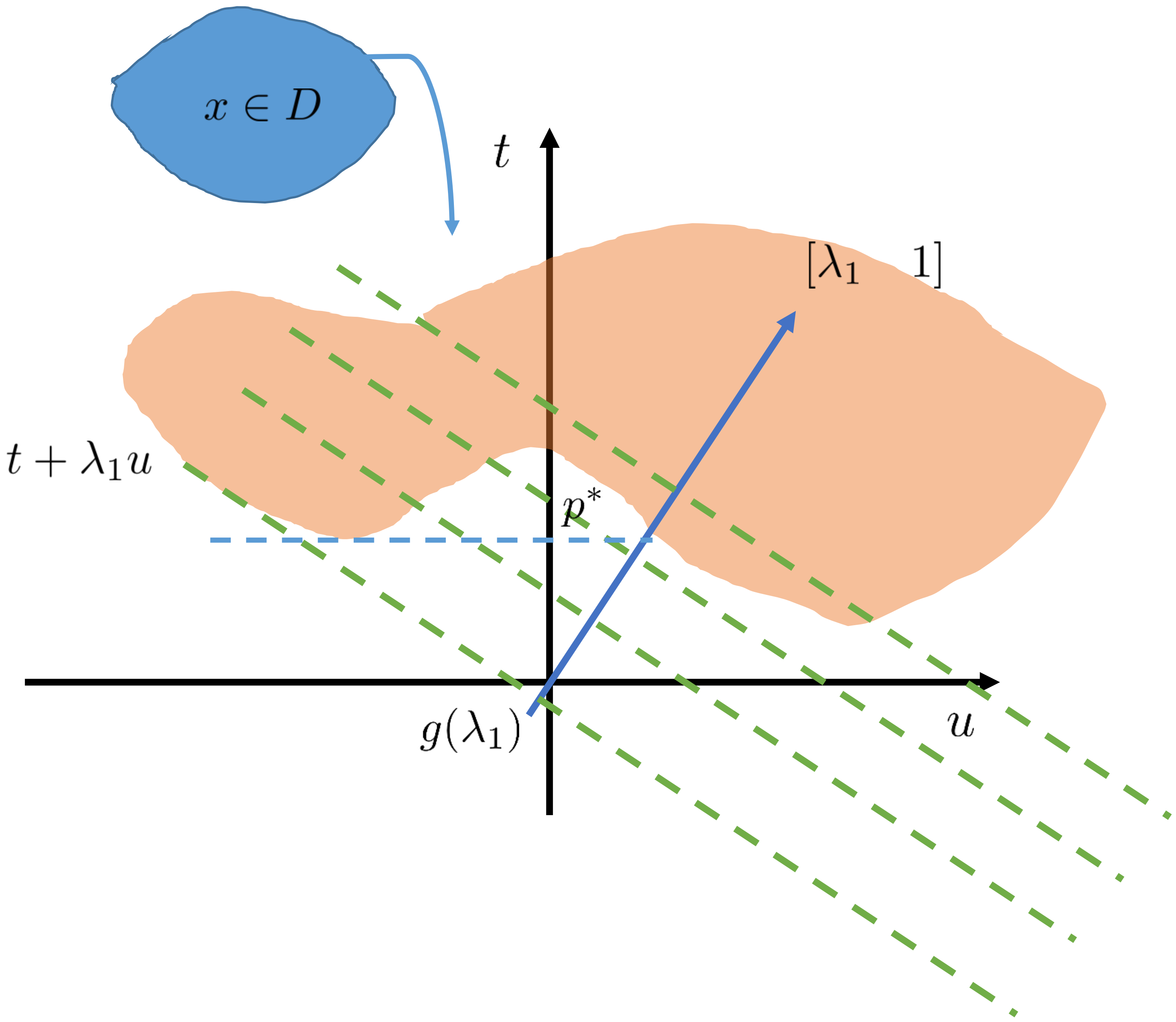

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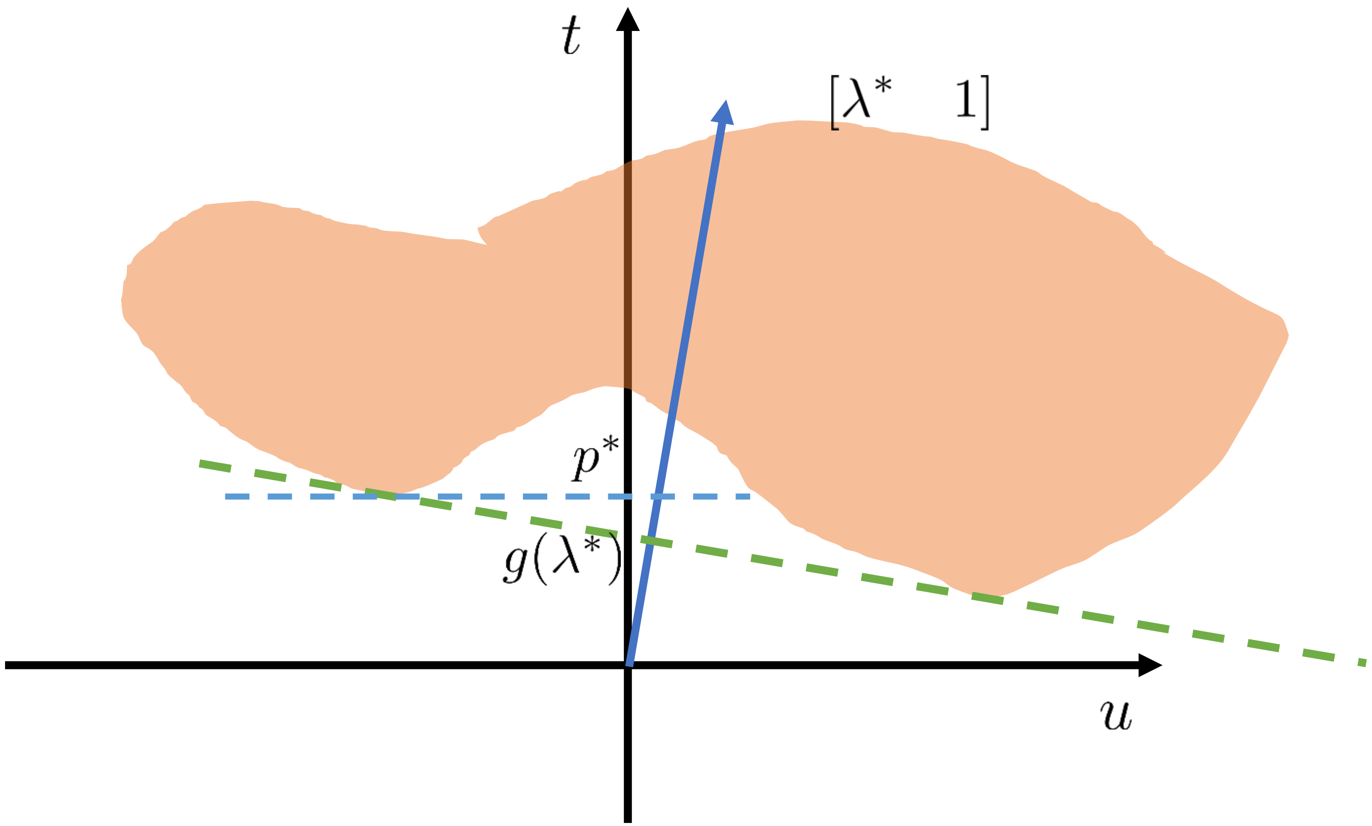
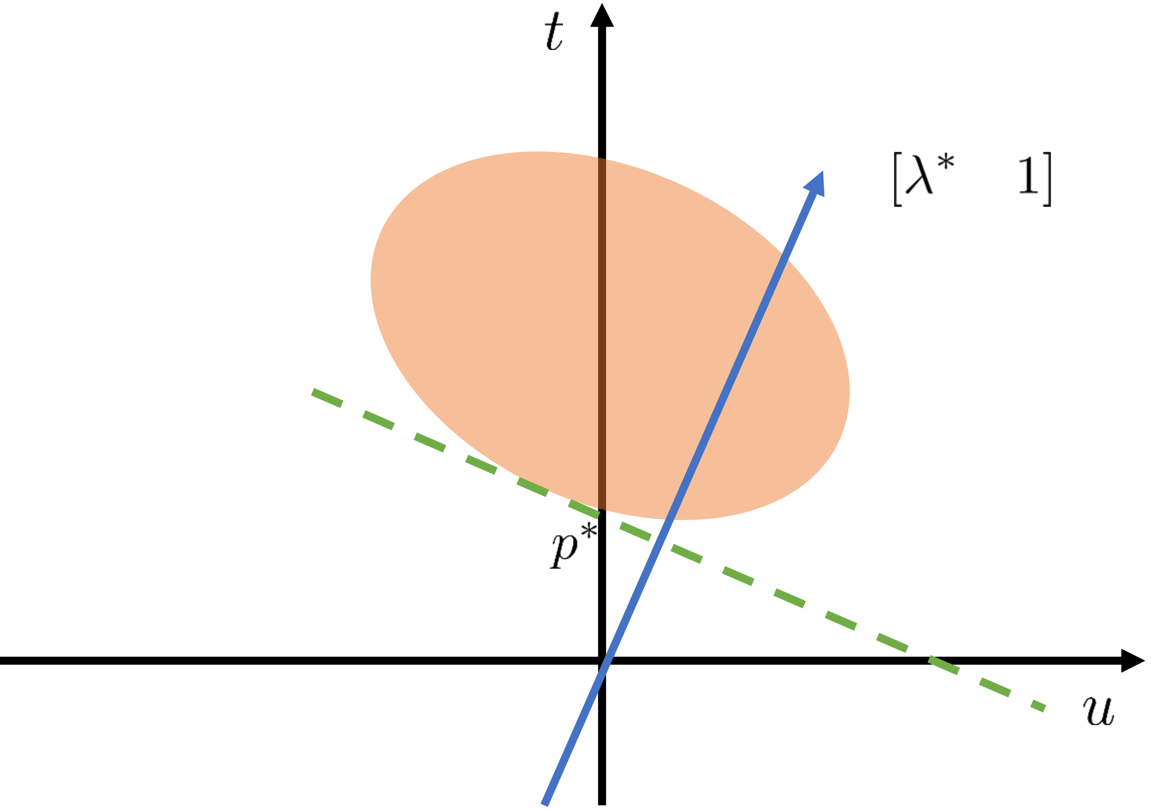


Duality – Complementary Slackness Geometric Interpretation

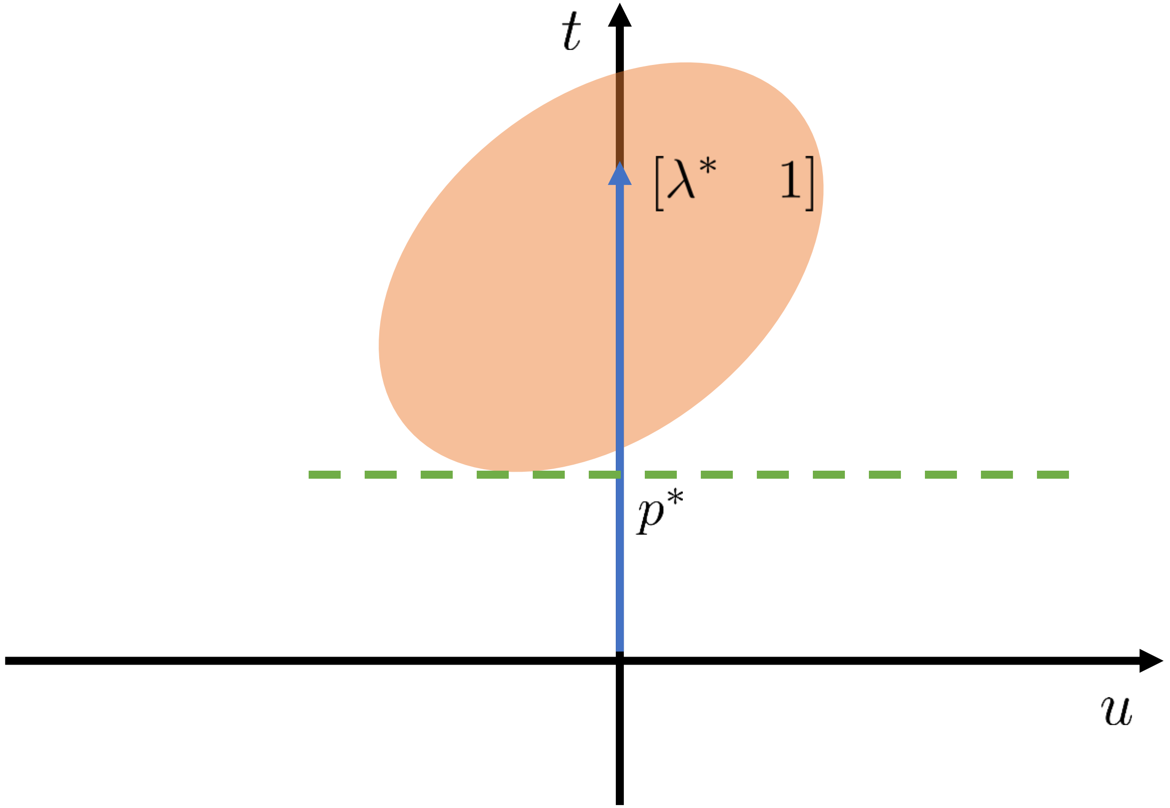
Problem 1:

$$\lambda^* > 0 \;\rightarrow\; f_1(x^*) = 0$$



Problem 2:

$$f_1(x^*) < 0 \;\rightarrow\; \lambda^* = 0$$



Feedback: Questions or Comments?

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Slides available at: https://www.zubairkhalid.org/ee563_2020.html

(Let me know should you need latex source)